EXPONENTIAL FUNCTIONS

Name:

EXPONENTS

Positive integer exponent means

repeated multiplication

Ex.

$x \cdot x \cdot x = x^3$

Negative exponent means

take the reciprocal of the base.

$x^{-3} = \frac{1}{x^3}$ $\frac{1}{x^{-2}} = x^2$

$\left(\frac{a}{b}\right)^{-4} = \left(\frac{b}{a}\right)^4$

Fractional exponent - denominator means

take the n^{th} root.

$x^{\frac{1}{n}} = \sqrt[n]{x}$

numerator means

repeated multiplication

$x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

Ex.

$x^{\frac{3}{4}} = \sqrt[4]{x^3} = \left(\sqrt[4]{x}\right)^3$

↑ no calc method

Move to the right to take the nth power →

Base below – *nth power to the right*	*2*	*3*	*4*	*5*	*6*
2	4	8	16	32	64
3	9	27	81	243	729
4	16	64	256	1024	4096
5	25	125	625	3125	15625
6	36	216	1296	7776	46656
7	49	343	2401	16807	117,649
8	64	512	4096	32768	262,144
9	81	729	6561	59049	531,441
10	100	1000	10000	100000	1,000,000

← *Move to the left to take the nth root*

Power Function:

$f(x) = ax^n$

↑ Variable is the base (not exponential)

Ex. $f(x) = 3x^2$

Exponential Function:

$f(x) = ab^x$

↖ Variable is the exponent

Ex. $f(x) = 3 \cdot 2^x$

Compound Interest

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

↓ also y

$A = y$ = final account balance

P = principle/initial

r = annual interest rate to use change % to decimal.

n = number of times interest is compounded per year

t = time in years

Exponent Rules/Properties

$a^m \cdot a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$(ab)^m = a^m b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$(a^m)^n = a^{mn}$

$a^0 = 1$

$a^1 = a$

$a^{-1} = \frac{1}{a}$

$(a)^{\frac{1}{n}} = \sqrt[n]{a}$ OR $\left(\sqrt[n]{a}\right)^1$

Scientific Notation

Form: $c \times 10^n$ $1 \leq |c| < 10$, n is an integer

In standard form is $|c| > 1$? Then exponent is positive.

In standard form is $0 < |c| < 1$? Then exponent is negative.

Standard	Scientific Notation
1,301	1.301×10^3
.00206	2.06×10^{-3}

Writing models: For table

Is it linear?

Do x's go up by 1?
Can use linear function if
difference
of consecutive y's is constant.

$y_2 - y_1 \quad (x_2 > x_1)$

check "y-pairs"
if constant $y_2 - y_1 = m$

use $y = mx + b$
(slope ↑, y-int. ↑)

Is it exponential?
Do x's go up by 1?

Can use exponential if ratio $\frac{y_2}{y_1}$
of consecutive y's is constant.

check "y-pairs"

if constant $\frac{y_2}{y_1} = b$

use $y = a \cdot b^x$
(y-int. ↑, base ↑)

What if I have a graph?
Use points from graph to find the base.

To Graph an Exponential Function

$y = ab^x$ ← this form works for all.

1. Identify parent function: $y = 1 \cdot b^x$

Ex. $y = 2^x$ $y = 3^x$ $y = (\frac{1}{2})^x$

2. Create a table with at least 3 key points.

Ex. $y = 2^x$

x	y
-1	$\frac{1}{b}$
0	1
1	b

$y = 2^x$

x	y
-1	$\frac{1}{2}$
0	1
1	2

$y = 3^x$

x	y
-1	$\frac{1}{3}$
0	1
1	3

$y = (\frac{1}{2})^x$

x	y
-1	2
0	1
1	$\frac{1}{2}$

3. Plot points and connect them with a smooth curve (don't forget arrows)

To compare a graph with its parent.

$y = ab^{cx+h} + k$

reflections: If a is negative → vertical reflection over the x-axis. $y = 3^{-x}$

If c is negative → horizontal reflection over the y-axis, $y = -3^x$

vertical stretch/compression:

$|a| > 1$ → vertical stretch $y = 3 \cdot 2^x$

$0 < |a| < 1$ → vertical compression $y = \frac{1}{3} \cdot 2^x$

horizontal stretch/compression:

$|c| > 1$ → horizontal compression $y = 2^{3x}$

$0 < |c| < 1$ → horizontal stretch $y = 2^{\frac{1}{3}x}$

vertical shifts:

$+k$ → shift up k $y = 2^x + 3$

$-k$ → shift down k $y = 2^x - 3$

horizontal shifts:

$+h$ → shift left h $y = 2^{x+3}$

$-h$ → shift right h $y = 2^{x-3}$

Types of Exponential Functions
(2 subsets)

Exponential Growth

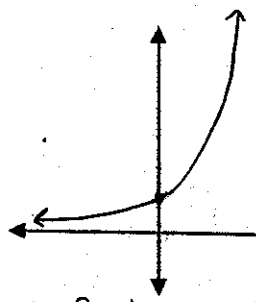
$y = a \cdot b^x \quad b > 1$

$y = a(1+r)^t$

a = initial amount

r = growth rate

t = time



Ex. Bacteria growth, Appreciation, Investments

Exponential Decay

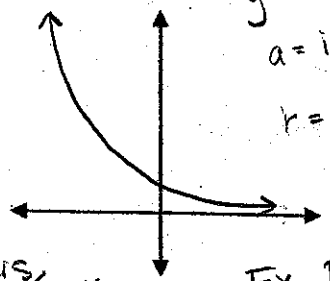
$y = a \cdot b^x \quad 0 < b < 1$

$y = a(1-r)^t$

a = initial amount

r = decay rate

t = time



Ex. Radioactive Decay, Cooling, Depreciation.

Compound interest (on other side) is a special type of exponential growth.