Name: Key 2018 - 2019 Period: ____________ Date: ________ ID: A

Algebra 1 Chapter 6 Exponentials Study Guide

Be sure to do 1 - 10 without a calculator.

____ 1. Which statements about function f are true?

left 3

$f(x) = \frac{5}{2}\left(\frac{9}{10}\right)^{x+3} + 4$ ← up 4

stretch by $2\frac{1}{2}$

a. The domain is all real numbers.

b. The graph of f is a translation 4 units right and 3 units up of the parent function, $g(x) = \left(\frac{9}{10}\right)^x$.

c. The graph of f is a vertical stretch by a factor of $\frac{5}{2}$ of the graph of the parent function, $g(x) = \left(\frac{9}{10}\right)^x$.

d. The range is $y > 4$.

e. The y-intercept of the graph of f is above the y-intercept of the graph of the parent function, $g(x) = \left(\frac{9}{10}\right)^x$.

← give #

$\boxed{\text{Evaluate}}$ the expression. Rewrite any expression containing fractional exponents in radical form before evaluating.

2. $(-5)^{-1} = \frac{1}{-5} = \frac{-1}{5}$

3. $\frac{(-6)^{-5}}{3^{-2}} = \frac{3^2}{(-6)^5} = \frac{9}{-7776} \overset{\div 9}{=} \frac{-1}{864}$

4. $\sqrt{9} = 3$

5. $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

6. $8^{\frac{7}{3}} = \left(\sqrt[3]{8}\right)^7 = 2^7 = 128$

7. $(-32)^{6/5} = \left(\sqrt[5]{-32}\right)^6 = (-2)^6 = 64$

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Evaluate the expression. Rewrite any expressions with fractional exponents in radical form before evaluating.

8. $(27)^{-2/3}$ $= \left(\frac{1}{27}\right)^{\frac{2}{3}} = \frac{(\sqrt[3]{1})^2}{(\sqrt[3]{27})^2} = \frac{1}{3^2} = \frac{1}{9}$

9. $-\sqrt[3]{-125}$ $= -(-5) = 5$

10. $(8)^{2/3} \bullet (27)^{-1/3}$ $= (\sqrt[3]{8})^2 \cdot \left(\frac{1}{27}\right)^{\frac{1}{3}} = 2^2 \cdot \sqrt[3]{\frac{1}{27}} = 4 \cdot \frac{1}{3} = \frac{4}{3}$

Evaluate the function for the given value of *x*. You may use your calculator for the rest of the study guide.

11. $y = 3^x; x = 2$ $3^2 = 9$

12. $f(x) = -4(4)^x; x = 2$ $f(2) = -4(4)^2$

$= -4(16) = -64$

Evaluate the function for the given value of *x*.

13. $f(x) = -2\left(\frac{1}{4}\right)^x; \; x = \frac{1}{2}$

$f(\tfrac{1}{2}) = -2\left(\frac{1}{4}\right)^{\frac{1}{2}} = -2 \cdot \sqrt{\frac{1}{4}} = -2 \cdot \frac{1}{2} = -1$

Find the indicated real *n*th root(s) of *a*.

14. $n = 3, a = 729$ $\sqrt[3]{729} = 9$

15. $n = 2, a = 16$ $\pm\sqrt[2]{16} = \pm 4$

16. $n = 2, a = -49$ $\sqrt[2]{-49}$ = no real roots

Simplify the expression. Write your answer using only positive exponents.

17. $(x^5)^9$ $= x^{45}$

18. $(-4t)^4$ $= (-4)^4 t^4 = 256t^4$

19. $(6k^3)^5$ $= 6^5 (k^3)^5 = 7776k^{15}$

20. $\dfrac{x^{-5}}{-10^{-3}y^{-1}}$ $= \dfrac{-10^3 y^1}{x^5} = \dfrac{-1000y}{x^5}$

21. $\dfrac{10^0 n^{-6} q^0}{4p^{-5}}$ $= \dfrac{1 \cdot 1 \cdot p^5}{4n^6} = \dfrac{p^5}{4n^6}$

Simplify the expression. Write your answer using only positive exponents.

22. $\dfrac{12x^{-5}y^3}{3^{-2}x^{-2}y^{-4}}$ $= \dfrac{12 \cdot 3^2}{1} \cdot \dfrac{x^2}{x^5} \cdot \dfrac{y^3 \cdot y^4}{1} = \dfrac{12 \cdot 9}{1} \cdot \dfrac{1}{x^3} \cdot \dfrac{y^7}{1} = \dfrac{108y^7}{x^3}$

23. $\left(5x^4y^0\right)^{-3}$ $= \left(\dfrac{1}{5x^4}\right)^3 = \dfrac{1^3}{5^3(x^4)^3} = \dfrac{1}{125x^{12}}$

24. $\left(-\dfrac{1}{2a^{-2}}\right)^{-3}$ $= \left(\dfrac{-1a^2}{2}\right)^{-3} = \left(\dfrac{-2}{a^2}\right)^3 = \dfrac{(-2)^3}{(a^2)^3} = \dfrac{-8}{a^6}$

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Determine whether the table represents a *linear function*, an *exponential growth function*, an *exponential decay function*, or *neither*. Be sure to show work to justify your answer. If the function is linear or exponential, write a model that represents the data.

25. Exponential decay

x's ↑ 1

x	0	1	2	3
y	81	27	9	3

$y_2 - y_1$

$3 - 9 = -6$

$9 - 27 = -18$

The difference is not constant so the function is not linear.

$\frac{y_2}{y_1}$

$\frac{3}{9} = \frac{1}{3}$

$\frac{9}{27} = \frac{1}{3}$

$\frac{27}{81} = \frac{1}{3}$

The ratio is constant so the function is exponential.

$y = a \cdot b^x$

$a = 81 \quad b = \frac{1}{3}$

$\boxed{y = 81\left(\frac{1}{3}\right)^x}$

Determine whether the table represents a *linear function*, an *exponential growth function*, an *exponential decay function*, or *neither*. Be sure to show work to justify your answer. If the function is linear or exponential, write a model that represents the data.

26. Exponential growth

x's ↑ 1

x	–1	0	1	2
y	0.25	0.75	2.25	6.75

$a = .75$

$y_2 - y_1$

$6.75 - 2.25 = 4.5$

$2.25 - .75 = 1.5$

Difference is not constant so the function is not linear.

$\frac{y_2}{y_1}$: $\frac{6.75}{2.25} = 3$

$\frac{2.25}{.75} = 3$

$\frac{.75}{.25} = 3$

The ratio is constant so the function is exponential.

$b = 3$

$y = .75(3)^x$

27.

x	–3	0	3	6
y	8	5	2	–1

y-int.

$y_2 - y_1$

$-1 - 2 = -3$

$2 - 5 = -3$

$5 - 8 = -3$

The difference is constant so the function is linear.

$y = -3x + 5$

Determine whether the function represents *exponential growth* or *exponential decay*. Identify the percent rate of change.

28. $y = 2(0.85)^t$ $\quad .85 < 1$ Exponential decay

$1 - r = .85$

$-1 \qquad -1$

$-r = -.15$

$r = .15$

15% decrease is the percent rate of change.

29. $y = 3\left(\frac{7}{5}\right)^t$ $\quad \frac{7}{5} > 1$ Exponential growth

$1 + r = \frac{7}{5}$

$-\frac{5}{5} \qquad -\frac{5}{5}$

$r = \frac{2}{5} = .4$

40% increase is the percent rate of change.

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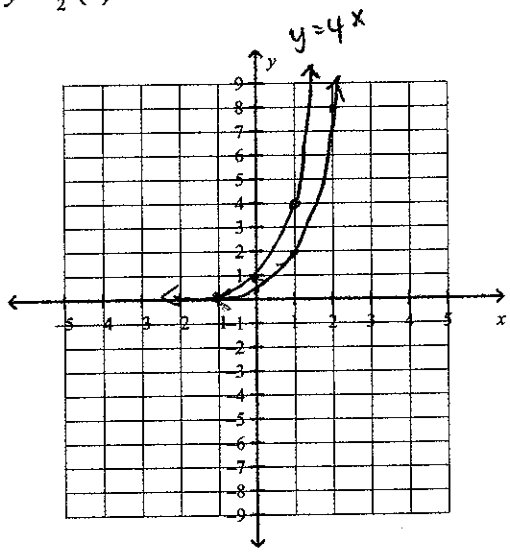
Graph the function given below and the associated parent function. Be sure to give a **table** for each. Describe the **domain and range** of the function below and describe the **transfomations** from the parent.

30. $y = \frac{1}{2}(4)^x$



parent

$y = 4^x$

x	y
-1	$\frac{1}{4}$
0	1
1	4

$y = 4^{-1}$

$y = \frac{1}{4}$

$y = \frac{1}{2}(4)^x$

x	y
-1	$\frac{1}{8}$
0	$\frac{1}{2}$
1	2
2	8

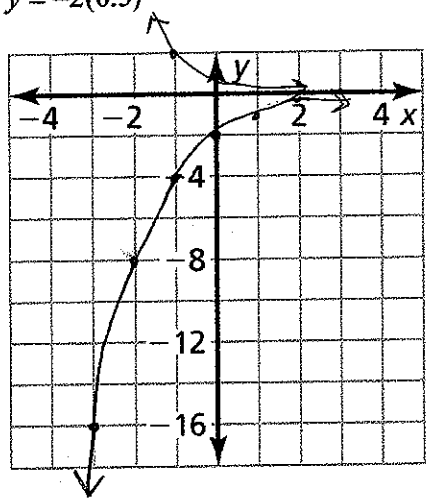
D: ℝ

R: $\{y > 0\}$

$y = \frac{1}{2}(4)^{-1} = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

$y = \frac{1}{2}(4)^x$ is a vertical compression of $y = (4)^x$.

31. $y = -2(0.5)^x$



parent

$y = (.5)^x = (\frac{1}{2})^x$

x	y
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

$y = -2(\frac{1}{2})^x$

x	y
-1	-4
0	-2
1	-1
2	$\frac{-1}{2}$
-2	-8
-3	-16

$y = -2(.5)^x$ is a reflection over the x-axis and a vertical stretch by a factor of 2 of $y = (.5)^x$

D: ℝ

R: $\{y < 0\}$

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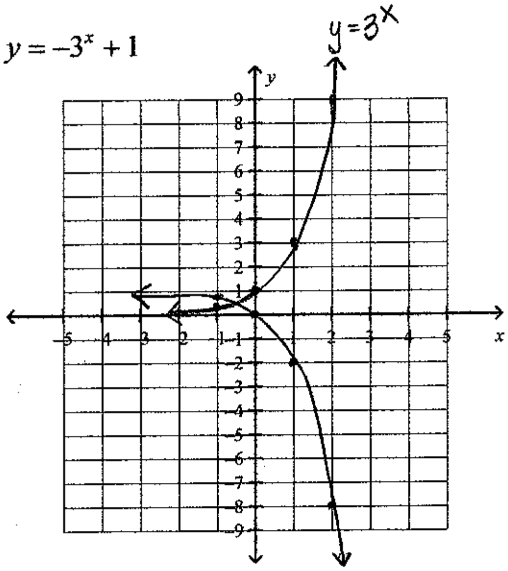
Graph the function given below and the associated parent function. Be sure to give a **table** for each. Describe the **domain and range** of the function below and describe the **transfomations** from the parent.

32. $y = -3^x + 1$



$y = 3^x$

x	y
-1	$\frac{1}{3}$
0	1
1	3
2	9

$y = -3^x + 1$

x	y
-1	$\frac{2}{3}$
0	0
1	-2
2	-8

D: $\mathbb{R}$

R: $\{y<1\}$

$y = -3^x + 1$ is a reflection over the x-axis and a shift up 1 of $y = 3^x$.

33. You deposit $675 in a savings account that earns 6% interest compounded monthly.

a. Write a function that represents the balance after t years.

$= P(1+\frac{r}{n})^{nt}$

$P = 675$ $r = .06$ $n = 12$

$y = 675\left(1 + \frac{.06}{12}\right)^{12t}$

$y = 675(1.005)^{12t}$

b. What is the balance after 3 years?

$y = 675(1.005)^{12(3)}$

$y = 675(1.005)^{36} = 807.76$

The account will contain $807.76 after three years.

34. Earth's mass is approximately 5.97×10^{24} kilograms. The table shows the masses of different planets. How many times bigger is the mass of Earth than the mass of Mercury? Write your answer in scientific notation and in standard form.

Planet	Mass (kilograms)
Mercury	3.30×10^{23}
Venus	4.87×10^{24}
Mars	6.42×10^{23}
Jupiter	1.90×10^{27}
Saturn	5.69×10^{26}
Uranus	8.68×10^{25}
Neptune	1.02×10^{26}

$\frac{\text{Earth}}{\text{Mercury}} = \frac{5.97 \times 10^{24}}{3.3 \times 10^{23}}$

$= \frac{5.97}{3.3} \cdot \frac{10^{24}}{10^{23}}$

$\approx 1.81 \times 10^1$ Scientific notation

≈ 18.1 Standard form

The Earth is 1.81×10^1 times larger in mass than Mercury.

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35. The bacteria *E. coli* often cause illness among people who eat infected food. Suppose that a **single** *E. coli* bacterium in a batch of ground beef begins doubling every minute.

$\cdot 2$

a. Complete the table below that represents the number of bacteria after x minutes. (Assume no bacteria die.)

Minutes, *x*	0	1	2	3	4	5	6
Number of bacteria, *y*	1	2	4	8	16	32	64

$1 \cdot 2 = 2$
$2 \cdot 2 = 4$
$4 \cdot 2 = 8$
$8 \cdot 2 = 16$
$16 \cdot 2 = 32$
$32 \cdot 2 = 64$

b. Write an equation that can be used to calculate the number of bacteria in the food after any number of minutes.

$a = 1$
$b = 2$

$y = 1 \cdot 2^x$

c. How many bacteria will there be after 20 minutes?

After 20 minutes there will 1,048,576 bacteria.

$y = 1 \cdot 2^{20}$

$y = 1{,}048{,}576$

Solve the equation.

36. $3^{4x} = 3^{2x+8}$

$4x = 2x + 8$
$-2x \quad -2x$
$2x = 8$
$\boxed{x = 4}$

37. $3^x = 9$

$3^x = 3^2$
$\boxed{x = 2}$

38. $100^{5x-4} = 10^{-x-1}$

$(10^2)^{5x-4} = 10^{-x-1}$
$10^{10x-8} = 10^{-x-1}$

$10x - 8 = -x - 1$
$+x \quad +x$
$11x - 8 = -1$
$+8 \quad +8$
$11x = 7$
$\boxed{x = \frac{7}{11}}$

Solve the equation. Check your solution.

39. $2 \bullet 4^{x+1} = \frac{1}{32}$

$2^1 \cdot (2^2)^{x+1} = (32)^{-1}$
$2^1 \cdot 2^{2x+2} = (2^5)^{-1}$
$2^{2x+3} = 2^{-5}$
$2x + 3 = -5$
$-3 \quad -3$
$\frac{2x}{2} = \frac{-8}{2}$

$\boxed{x = -4}$