Chapter 8

Maintaining Mathematical Proficiency

50. The function q is of the form $y = f(x - h)$, where $h = -6$. So, the graph of q is a horizontal translation 6 units left of the graph of f.

51. The function h is of the form $y = -af(x)$, where $a = 0.5$. So, the graph of h is a vertical shrink by a factor of 0.5 and a reflection in the x-axis of the graph of f.

52. The function g is of the form $y = f(x - h) + k$, where $h = 2$ and $k = 5$. So, the graph of g is horizontal translation 2 units right and a vertical translation 5 units up of the graph of f.

53. The function p is of the form $y = af(x - h)$, where $a = 3$ and $h = -1$. So, the graph of p is a vertical stretch by a factor of 3 and a horizontal translation 1 unit left of the graph of f.

8.1–8.3 What Did You Learn? (p. 439)

1. *Sample answer:* Because the highest point has a y-coordinate of 0, the height is the opposite of the y-coordinate of the lowest points. The width is the absolute value of the difference of the x-coordinates of the endpoints.

2. *Sample answer:* The t-intercept of the graph is the total time before the water balloon hits the ground.

3. *Sample answer:* Use the definition of vertex to identify $f\left(-\frac{b}{2a}\right)$ as the y-coordinate of the vertex. Then use the definition of maximum value/minimum value to recognize that this is also the maximum or minimum value of the function.

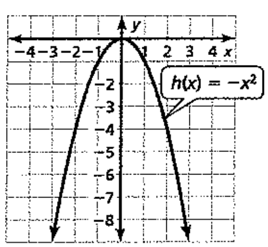
8.1–8.3 Quiz (p. 440) ← Start here

1. The vertex is $(1, 4)$. The axis of symmetry is $x = 1$. The domain is all real numbers. The range is $y \leq 4$. When $x < 1$, y increases as x increases. When $x > 1$, y increases as x decreases.

2. The vertex is $(-2, 5)$. The axis of symmetry is $x = -2$. The domain is all real numbers. The range is $y \geq 5$. When $x < -2$, y increases as x decreases. When $x > -2$, y increases as x increases.

3.

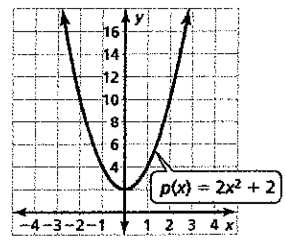
x	−2	−1	0	1	2
$h(x)$	−4	−1	0	−1	−4



The graph of h is a reflection in the x-axis of the graph of f.

4.

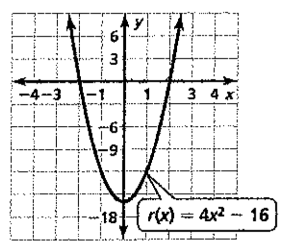
x	−2	−1	0	1	2
$p(x)$	10	4	2	4	10



The graph of p is a vertical stretch by a factor of 2 and a vertical translation 2 units up of the graph of f.
shift

5.

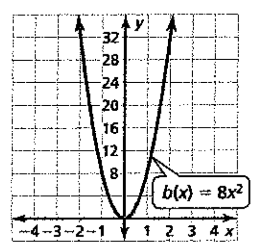
x	−2	−1	0	1	2
$r(x)$	0	−12	−16	−12	0



The graph of r is a vertical stretch by a factor of 4 and a vertical translation 16 units down of the graph of f.
shift

6.

x	−2	−1	0	1	2
$b(x)$	32	8	0	8	32

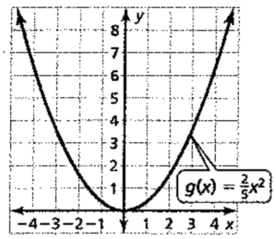


The graph of b is a vertical stretch by a factor of 8 of the graph of f.

 Book quiz 8.1-8.3

7.

x	-4	-2	0	2	4
$g(x)$	6.4	1.6	0	1.6	6.4

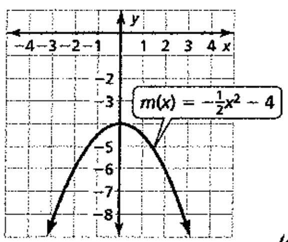


compression

The graph of g is a vertical shrink by a factor of $\frac{2}{5}$ of the graph of f.

8.

x	-2	-1	0	1	2
$m(x)$	-6	-4.5	-4	-4.5	-6



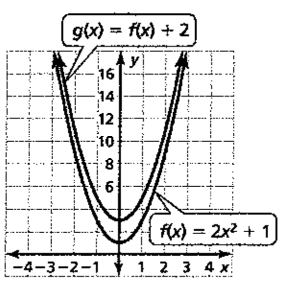
compression

The graph of m is a vertical shrink by a factor of $\frac{1}{2}$, a reflection in the x-axis, and a vertical translation 4 units down of the graph of f.

shift

9. The graph of g is a vertical translation 2 units up of the graph of f.

x	-2	-1	0	1	2
$f(x) = 2x^2 + 1$	9	3	1	3	9
$g(x) = f(x) + 2$	11	5	3	5	11



$g(x) = f(x) + 2$

$g(x) = (2x^2 + 1) + 2$

$= 2x^2 + (1 + 2)$

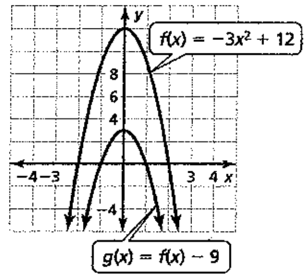
$= 2x^2 + 3$

So, $g(x) = 2x^2 + 3$.

shift

10. The graph of g is a vertical translation 9 units down of the graph of f.

x	-2	-1	0	1	2
$f(x) = -3x^2 + 12$	0	9	12	9	0
$g(x) = f(x) - 9$	-9	0	3	0	-9



$g(x) = f(x) - 9$

$g(x) = (-3x^2 + 12) - 9$

$= -3x^2 + (12 - 9)$

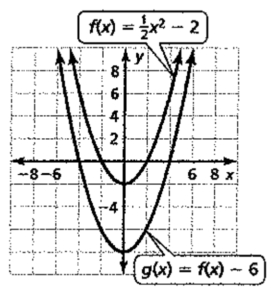
$= -3x^2 + 3$

So, $g(x) = -3x^2 + 3$.

Shift

11. The graph of g is a vertical translation 6 units down of the graph of f.

x	-4	-2	0	2	4
$f(x) = \frac{1}{2}x^2 - 2$	6	0	-2	0	6
$g(x) = f(x) - 6$	0	-6	-8	-6	0



$g(x) = f(x) - 6$

$= \left(\frac{1}{2}x^2 - 2\right) - 6$

$= \frac{1}{2}x^2 + (-2 - 6)$

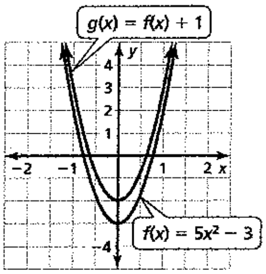
$= \frac{1}{2}x^2 - 8$

So, $g(x) = \frac{1}{2}x^2 - 8$.

Chapter 8

12. The graph of g is a vertical translation 1 unit up of the graph of f.

x	-1	-0.5	0	0.5	1
$f(x) = 5x^2 - 3$	2	-1.75	-3	-1.75	2
$g(x) = f(x) + 1$	3	-0.75	-2	-0.75	3



$g(x) = f(x) + 1$

$g(x) = (5x^2 - 3) + 1$

$= 5x^2 + (-3 + 1)$

$= 5x^2 - 2$

So, $g(x) = 5x^2 - 2$.

13. The axis of symmetry is $x = -\frac{b}{2a} = -\frac{(-4)}{2(-4)} = \frac{4}{-8} = -\frac{1}{2}$.

$f(x) = -4x^2 - 4x + 7$

$f\left(-\frac{1}{2}\right) = -4\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) + 7$

$= -4\left(\frac{1}{4}\right) + 2 + 7$

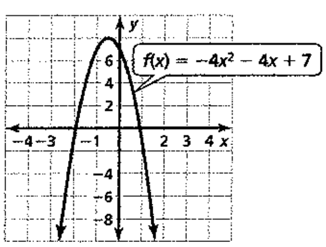
$= -1 + 2 + 7$

$= 1 + 7$

$= 8$

So, the vertex is $\left(-\frac{1}{2}, 8\right)$.

The y-intercept is 7. So, the points $(0, 7)$ and $(-1, 7)$ lie on the graph.



The domain is all real numbers. The range is $y \leq 8$.

14. The axis of symmetry is $x = -\frac{b}{2a} = -\frac{12}{2(2)} = \frac{-12}{4} = -3$.

$f(x) = 2x^2 + 12x + 5$

$f(-3) = 2(-3)^2 + 12(-3) + 5$

$= 2(9) - 36 + 5$

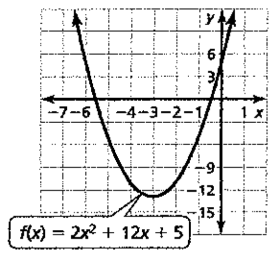
$= 18 - 36 + 5$

$= -18 + 5$

$= -13$

So, the vertex is $(-3, -13)$.

The y-intercept is 5. So, the points $(0, 5)$ and $(-6, 5)$ lie on the graph.



The domain is all real numbers. The range is $y \geq -13$.

15. The axis of symmetry is $x = -\frac{b}{2a} = -\frac{4}{2(1)} = \frac{-4}{2} = -2$.

$y = x^2 + 4x - 5$

$y = (-2)^2 + 4(-2) - 5$

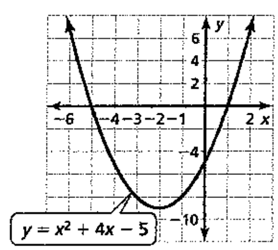
$= 4 - 8 - 5$

$= -4 - 5$

$= -9$

So, the vertex is $(-2, -9)$.

The y-intercept is -5. So, the points $(0, -5)$ and $(-4, -5)$ lie on the graph.



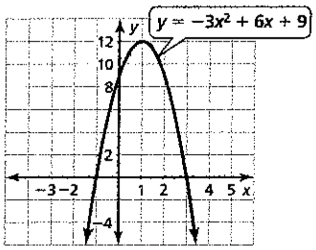
The domain is all real numbers. The range is $y \geq -9$.

16. The axis of symmetry is $x = -\frac{b}{2a} = -\frac{6}{2(-3)} = \frac{-6}{-6} = 1$.

$y = -3x^2 + 6x + 9$
$y = -3(1)^2 + 6(1) + 9$
$= -3(1) + 6 + 9$
$= -3 + 6 + 9$
$= 3 + 9$
$= 12$

So, the vertex is (1, 12).

The y-intercept is 9. So, the points (0, 9) and (2, 9) lie on the graph.



The domain is all real numbers. The range is $y \leq 12$.

17. For $f(x) = 5x^2 + 10x - 3$, $a = 5$ and $5 > 0$. So, the parabola opens up, and the function has a minimum value.

$x = -\frac{b}{2a} = -\frac{10}{2(5)} = \frac{-10}{10} = -1$

$f(x) = 5x^2 + 10x - 3$
$f(-1) = 5(-1)^2 + 10(-1) - 3$
$= 5(1) - 10 - 3$
$= 5 - 10 - 3$
$= -5 - 3$
$= -8$

The minimum value is -8.

18. For $f(x) = -\frac{1}{2}x^2 + 2x + 16$, $a = -\frac{1}{2}$ and $-\frac{1}{2} < 0$. So, the parabola opens down, and the function has a maximum value.

$x = -\frac{b}{2a} = -\frac{2}{2\left(-\frac{1}{2}\right)} = \frac{-2}{-1} = 2$

$f(x) = -\frac{1}{2}x^2 + 2x + 16$
$f(2) = -\frac{1}{2}(2)^2 + 2(2) + 16$
$= -\frac{1}{2}(4) + 4 + 16$
$= -2 + 4 + 16$
$= 2 + 16$
$= 18$

The maximum value is 18.

19. For $y = -x^2 + 4x + 12$, $a = -1$ and $-1 < 0$. So, the parabola opens down, and the function has a maximum value.

$x = -\frac{b}{2a} = -\frac{4}{2(-1)} = \frac{-4}{-2} = 2$

$y = -x^2 + 4x + 12$
$y = -(2)^2 + 4(2) + 12$
$= -4 + 8 + 12$
$= 4 + 12$
$= 16$

The maximum value is 16.

20. For $y = 2x^2 + 8x + 3$, $a = 2$ and $2 > 0$. So, the parabola opens up, and the function has a minimum value.

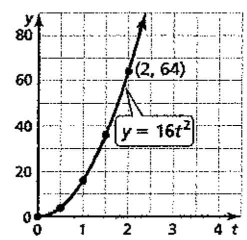
$x = -\frac{b}{2a} = -\frac{8}{2(2)} = -\frac{8}{4} = -2$

$y = 2x^2 + 8x + 3$
$y = 2(-2)^2 + 8(-2) + 3$
$= 2(4) - 16 + 3$
$= 8 - 16 + 3$
$= -8 + 3$
$= -5$

The minimum value is -5.

21.

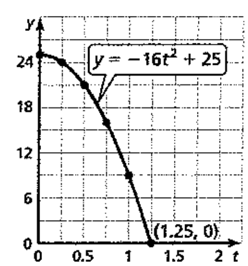
t	0	0.5	1	1.5	2
y	0	4	16	36	64



The point (2, 64) lies on the graph. So, it takes 2 seconds for the coconut to fall 64 feet.

22. a.

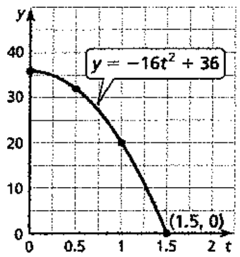
t	0	0.25	0.5	0.75	1	1.25
y	25	24	21	16	9	0



The positive t-intercept is 1.25. So, the pinecone hits the ground after 1.25 seconds.

b. $y = -16t^2 + 36$

t	0	0.5	1	1.5
y	36	32	20	0



The positive t-intercept of the graph that represents the height of the second pinecone is 1.5. So, the second pinecone hits the ground after 1.5 seconds, which means the first pinecone hits the ground in the least amount of time.

23. $t = -\frac{b}{2a} = -\frac{32}{2(-16)} = \frac{-32}{-32} = 1$

$h(t) = -16t^2 + 32t + 2$

$h(1) = -16(1)^2 + 32(1) + 2$

$= -16(1) + 32 + 2$

$= -16 + 32 + 2$

$= 16 + 2$

$= 18$

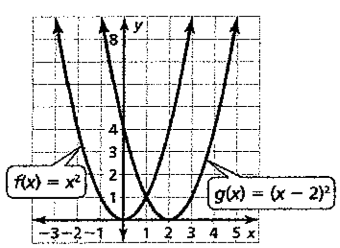
Because the midpoint of the graph occurs when $x = 1$, the domain is $0 \le t \le 2$. The highest point is the vertex (1, 18), and the lowest points are (0, 0) and (2, 0). So, the range is $0 \le h \le 18$. The maximum height of the softball is 18 feet.

End of answers for book quiz 8.1-8.3

8.4 Explorations (p. 441)

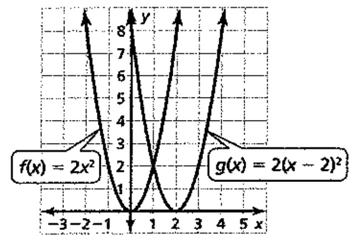
1. a.

x	−2	−1	0	1	2	3	4
$f(x) = x^2$	4	1	0	1	4	9	16
$g(x) = (x - 2)^2$	16	9	4	1	0	1	4



b.

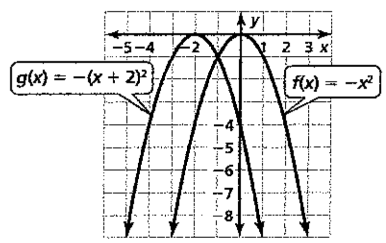
x	−2	−1	0	1	2	3	4
$f(x) = 2x^2$	8	2	0	2	8	18	32
$g(x) = 2(x - 2)^2$	32	18	8	2	0	2	8



Sample answer: The value of h causes a horizontal translation of the graph of $y = ax^2$.

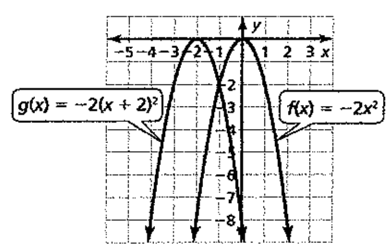
2. a.

x	−4	−3	−2	−1	0	1	2
$f(x) = -x^2$	−16	−9	−4	−1	0	−1	−4
$g(x) = -(x + 2)^2$	−4	−1	0	−1	−4	−9	−16



b.

x	−4	−3	−2	−1	0	1	2
$f(x) = -2x^2$	−32	−18	−8	−2	0	−2	−8
$g(x) = -2(x + 2)^2$	−8	−2	0	−2	−8	−18	−32



Sample answer: The value of h causes a horizontal translation of the graph of $y = ax^2$.

3. When $h > 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation h units to the right of the graph of $f(x) = ax^2$. When $h < 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation $|h|$ units to the left of the graph of $f(x) = ax^2$.