9.4 Notetaking with Vocabulary

Perfect square trinomial

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)^2 = a^2 + 2ab + b^2$

Success Criteria

I can complete the square. * Day 1

I can solve quadratic equations by completing the square.

I can find and use the maximum and minimum values of a quadratic function.

I can analyze and interpret the equation and/or graph of a quadratic.

Fits the square-double-square pattern.

Core Concepts

"Completing the Square" – the process of adding a constant "c" to the expression "$x^2 + bx$" so that $x^2 + bx + c$ is perfect square trinomial (fits square-double-square).

Completing the Square

a must be 1

Words To complete the square for an expression of the form $x^2 + bx$, follow these steps.

Step 1 Divide "b" by 2 / Take $\frac{1}{2}$ of b, the coefficient of x. (go in the squared binomial)

Step 2 Square the result from Step 1. (new "c")

Step 3 Add the result from Step 2 to $x^2 + bx.$ + c (perfect square trinomial)

Step 4 Factor the resulting expression as the square of binomial.

Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

In Exercises 1–6, ① complete the square for the expression. ② Then factor the trinomial.

1. $x^2 + 12x + c$ c?

sq. double sq.

$b = 12 \quad \frac{b}{2} = \frac{12}{2} = 6$

$6^2 = 36 \leftarrow$ new c

$x^2 + 12x + 36$

$= (x+6)^2$

2. $x^2 - 14x$

$b = -14$ ① $\frac{-14}{2} = \frac{b}{2} = -7$ save for later

② $(-7)^2 = 49 \leftrightarrow$ new "c"

③ $x^2 - 14x + 49$

④ $(x-7)^2$

3. $x^2 + 4x$

$b = 4$ ① $\frac{b}{2} = \frac{4}{2} = 2$

② $(2)^2 = 4$

③ $x^2 + 4x + 4$

④ $(x + 2)^2$

4. $x^2 + 18x$

$b = 18$ $\frac{18}{2} = 9$

$9^2 = 81$

$x^2 + 18x + 81$

$(x + 9)^2$

5. $x^2 - 7x$ ① $b = -7$ $\frac{b}{2} = \frac{-7}{2}$

② $\left(\frac{-7}{2}\right)^2 = \frac{49}{4}$

③ $x^2 - 7x + \frac{49}{4}$

④ $\left(x - \frac{7}{2}\right)^2$

6. $x^2 + 11x$

$b = 11$ $\frac{11}{2} = \frac{b}{2}$

$\left(\frac{11}{2}\right)^2 = \frac{121}{4}$

$x^2 + 11x + \frac{121}{4}$

$\left(x + \frac{11}{2}\right)^2$

Notes

When is it easy to complete the square? When b is even. (no fractions)

When is it difficult to complete the square? When b is odd.

Solving Quadratic Equations using Completing the Square

The method of completing the square can be used to solve ANY quadratic equation.

To solve a quadratic equation using this method you must first ~~right~~ write the equation in the form

$$x^2 + bx = d$$

Then complete the square of the left hand side – being sure to follow the properties of equality!!

In other words,

do the same thing to both sides of the equation.

After completing the square, your equation should have the form

$$\left(x + \frac{b}{2}\right)^2 = d + \left(\frac{b}{2}\right)^2 \qquad (x+4)^2 = 12$$

Then you finish solving by "taking the square root"

Give precise answer first.

In Exercises 7–13, solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

7. $x^2 - 8x = -15$

$b = -8$

$\frac{-8}{2} = -4$

$(-4)^2 = 16$ ← new c

$x^2 - 8x + 16 = -15 + 16$

$\sqrt{(x-4)^2} = \pm\sqrt{1}$

$x - 4 = \pm 1$

$+4 \quad +4$

$x = 4 \pm 1$

$x = 5$

$x = 3$

8. $x^2 + 2x = 3$

9. $x^2 - 26x = -9$

$b = -26$

$\frac{-26}{2} = -13$

$(-13)^2 = 169$

$x^2 - 26x + 169 = -9 + 169$

$\sqrt{(x-13)^2} = \pm\sqrt{160}$

$x - 13 = \pm\sqrt{160}$

$x - 13 = \pm\sqrt{16} \cdot \sqrt{10}$

$x - 13 = \pm 4\sqrt{10}$

$x = 13 \pm 4\sqrt{10}$

$x = 13 + 4\sqrt{10} \approx 25.65$

$x = 13 - 4\sqrt{10} \approx .35$

10. $x^2 - 12x = 10$

11. $x^2 - 12x + 9 = 0$

$-9 \quad -9$

$x^2 - 12x = -9$

$b = -12$

$\frac{-12}{2} = -6$

$(-6)^2 = 36$

$x^2 - 12x + 36 = -9 + 36$

$\sqrt{(x-6)^2} = \pm\sqrt{27}$

$x - 6 = \pm\sqrt{27}$

$x - 6 = \pm\sqrt{9} \cdot \sqrt{3}$

$x - 6 = \pm 3\sqrt{3}$

$x = 6 \pm 3\sqrt{3}$

$x = 6 + 3\sqrt{3} \approx 11.20$

$x = 6 - 3\sqrt{3} \approx .80$

12. $x^2 + 14x - 10 = 0$

13. $3x^2 + 6x - 1 = 0$

$3x^2+6x = 1$

$\frac{3(x^2+2x)}{3} = \frac{1}{3}$

$x^2+2x = \frac{1}{3}$

$x^2+2x+1 = \frac{1}{3}+1$

$(x+1)^2 = \frac{4}{3}$

$b=2$

$\frac{2}{2} = 1$

$(1)^2 = 1$

$\sqrt{(x+1)^2} = \pm\sqrt{\frac{4}{3}}$

$x+1 = \frac{\pm\sqrt{4}}{\sqrt{3}}$

$x+1 = \frac{\pm 2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x+1 = \frac{\pm 2\sqrt{3}}{3}$

$x = -1 \pm \frac{2\sqrt{3}}{3} = \frac{-3 \pm 2\sqrt{3}}{3}$

$x = \frac{(-3+2\sqrt{3})}{3} \approx .15$

$x = \frac{(-3-2\sqrt{3})}{3} \approx -2.15$

Vertex Form Another reason to complete the square –

to put quadratic in vertex form

$y = a(x-h)^2 + k$

(h,k)

Why?

- to find max/min → k
- to graph quadratic using transformations

The process is very similar except you will have a y in place of zero.

a>0 ↑ minimum

In Exercises 14–17, determine whether the quadratic function has a maximum or minimum value by putting it in vertex form. Then find the value. Determine the transformations of the parent function.

14. $y = -x^2 + 4x + 3$ ↓↻ Max

-3 -3

$y-3 = -x^2+4x$

$y-3 = -1(x^2-4x)$

$y-3-4 = -1(x^2-4x+4)$

$y-7 = -1(x-2)^2$

$y = -1(x-2)^2+7$

vertex (2,7) Max = 7

$b=-4$

$-\frac{4}{2} = -2$ go in the binomial

$(-2)^2 = 4$ new "c"

$-1(x^2-4x+4)$

$-x^2+4x-4$

1. shift up 7
2. shift right 2
3. reflection over x-axis

15. $y = x^2 + 6x + 10$

16. $y = 3x^2 + 3x - 1$

17. $y = -4x^2 + 8x + 12$

↓ $a = -4$ Max

$-12 \qquad -12$

$y - 12 = -4x^2 + 8x$ $\qquad b = -2$

$y - 12 = -4(x^2 - 2x)$ $\qquad \frac{-2}{2} = -1$

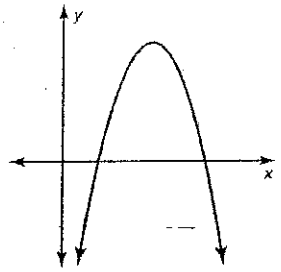
$y - 12 - 4 = -4(x^2 - 2x + 1)$ $\qquad (-1)^2 = 1$

$y - 16 = -4(x-1)^2$

$y = -4(x-1)^2 + 16$ $\qquad (1, 16)$

Max = 16

18. Which of the functions could be represented by the graph?



① Reflection over x-axis

② shift right 1

③ shift up 16

④ vertical stretch by a factor 4

must be one of these.

$f(x) = -\frac{1}{2}(x+4)^2 + 8$ X b/c vertex $(-4, 8)$ which would be in quadrant II.

$g(x) = -(x-5)^2 + 9$

$m(x) = (x-3)(x-12)$ X b/c $a > 0$ so opens up

$p(x) = -(x-2)(x-8)$

19. A diver jumps off a diving board. The function $h = -16x^2 + 6x + 5$ represents the height (in feet) of the diver after x seconds. What is the maximum height above the water of the diver? How many seconds did it take for the diver to reach the maximum height? Round your answers to the nearest hundredth.

Use old method to find vertex.

$a = -16 \quad b = 6 \quad c = 5$

$x = \frac{-b}{2a} = \frac{-6}{2(-16)} = \frac{-6}{-32} = \frac{3}{16} = .1875 \approx .19$

$h = -16\left(\frac{3}{16}\right)\left(\frac{3}{16}\right) + 6\left(\frac{3}{16}\right) + 5$

$= \frac{-9}{16} + \frac{18}{16} + \frac{80}{16} = \frac{89}{16} = 5.5625 \approx 5.56$

The diver reaches his maximum height after about .19 seconds. His maximum height is a little over $5\frac{1}{2}$ feet.