Name ______________________ Date ________

9.1 Notetaking with Vocabulary

Success Criteria

I can simplify expressions using properties of radicals.
I can simplify expressions by rationalizing the denominator.
I can perform operations with radicals.

$\underbrace{\sqrt[3]{x}}_{\text{radical}}$

Using Properties of Radicals

A radical expression is an expression that contains a radical. An expression involving a radical with index n is in simplest form when these three conditions are met.

1. No radicals have perfect n^{th} powers as factors (other than 1).
2. No radicals contain fractions.
3. No radicals appear in the denominator of a fraction.

$\frac{36 \div 9}{27 \div 9} = \frac{4}{3}$ $\qquad$ $\frac{36 \div 3}{27 \div 3} = \frac{12 \div 3}{9 \div 3} = \frac{4}{3}$

Product Property of Square Roots

Words The square root of a product equals the product of the square roots of the factors.

Numbers $\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Algebra $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256

Example: Simplify the expressions.

1. $\sqrt[2]{24}$

$\sqrt{4 \cdot 6}$

$\sqrt{4} \cdot \sqrt{6}$

$2\sqrt{6}$

2. $-\sqrt{48}$

$-\sqrt{16 \cdot 3}$

$-\sqrt{16} \cdot \sqrt{3}$

$-4\sqrt{3}$

3. $\sqrt{162g^6}$

$\sqrt{81 \cdot 2 \cdot g^6}$

$\sqrt{81} \cdot \sqrt{2} \cdot \sqrt{g^6}$

$9 \cdot \sqrt{2} \cdot g^3$

$9g^3\sqrt{2}$

4. $-\sqrt{512h^7}$

$-\sqrt{256 \cdot 2 \cdot h^6 \cdot h^1}$

$-\sqrt{256} \cdot \sqrt{2} \cdot \sqrt{h^6} \cdot \sqrt{h^1}$

$-16 \cdot \sqrt{2} \cdot h^3 \cdot \sqrt{h^1}$

$-16h^3\sqrt{2h}$

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9.1 **Notetaking with Vocabulary** (continued)

Quotient Property of Square Roots

Words The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

Numbers $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$ **Algebra** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

Examples: Simplify the expression.

5. $\sqrt{\frac{25}{64}} = \frac{\sqrt{25}}{\sqrt{64}} = \frac{5}{8}$

6. $-\sqrt{\frac{6}{49}} = \frac{-\sqrt{6}}{\sqrt{49}} = \frac{-\sqrt{6}}{7}$

7. $-\sqrt{\frac{196}{r^4}} = \frac{-\sqrt{196}}{\sqrt[2]{r^4}} = \frac{-14}{r^2}$

8. $\sqrt{\frac{49x^3}{64y^2}} = \frac{\sqrt{49x^3}}{\sqrt{64y^2}} = \frac{\sqrt{49}\cdot\sqrt{x^2}\cdot\sqrt{x^1}}{\sqrt{64}\cdot\sqrt{y^2}} = \frac{7x\sqrt{x}}{8y}$

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1{,}000$

You can extend the Product and Quotient Properties of Square Roots to other radicals such as cube roots. When using properties of cube roots, the radicands may contain negative numbers.

9. $\sqrt[3]{-135} = \sqrt[3]{-27\cdot 5} = \sqrt[3]{-27}\cdot\sqrt[3]{5} = -3\sqrt[3]{5}$

10. $\sqrt[3]{729} = 9$

11. $-\sqrt[3]{-192x^5} = -\sqrt[3]{-64\cdot 3\cdot x^3\cdot x^2} = -\sqrt[3]{-64}\cdot\sqrt[3]{3}\cdot\sqrt[3]{x^3}\cdot\sqrt[3]{x^2} = -(-4)\cdot\sqrt[3]{3}\cdot x\cdot\sqrt[3]{x^2} = 4x\sqrt[3]{3x^2}$

12. $\sqrt[3]{\frac{12a^6}{512b^4}}$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\sqrt{\frac{2}{5}} = \frac{\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{\sqrt{25}} = \frac{\sqrt{10}}{5}$

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9.1 Notetaking with Vocabulary (continued)

"well chosen 1"

When a radical is in the denominator of a fraction, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called rationalizing the denominator.

Simplify the expression.

13. $\frac{\sqrt{15}}{\sqrt{500}}$

$= \sqrt{\frac{15}{500}}$

$= \sqrt{\frac{3}{100}}$

$= \frac{\sqrt{3}}{\sqrt{100}} = \frac{\sqrt{3}}{10}$

14. $\sqrt{\frac{8 \div 4}{100 \div 4}}$

$= \sqrt{\frac{2}{25}}$

$= \frac{\sqrt{2}}{\sqrt{25}}$

$= \frac{\sqrt{2}}{5}$

15. $\frac{\sqrt{3x^2y^3}}{\sqrt{80xy^3}}$

$= \sqrt{\frac{3x^{\cancel{2}}\cancel{y^3}}{80\cancel{x}\cancel{y^3}}}$

$= \sqrt{\frac{3x}{80}}$

$= \frac{\sqrt{3x}}{\sqrt{80}} = \frac{\sqrt{3x}}{\sqrt{16 \cdot 5}} = \frac{\sqrt{3x}}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15x}}{4 \cdot 5}$

$\sqrt{16} \cdot \sqrt{5}$

$= \frac{\sqrt{15x}}{20}$

16. $\frac{8}{\sqrt[3]{16}} = \frac{8}{\sqrt[3]{8 \cdot 2}} = \frac{8}{\sqrt[3]{8} \cdot \sqrt[3]{2}}$

$= \frac{\cancel{8}^{4}}{\cancel{2}_{1}\sqrt[3]{2}} = \frac{4 \cdot \sqrt[3]{4}}{\sqrt[3]{2}\sqrt[3]{4}} \frac{4\sqrt[3]{4}}{\sqrt[3]{8}}$

$= \frac{\cancel{4}^{2}\sqrt[3]{4}}{\cancel{2}_{1}}$

$= 2\sqrt[3]{4}$

The binomials $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ where *a, b, c,* and *d* are rational numbers, are called conjugates. You can use conjugates to simplify radical expressions that contain a sum or difference involving square roots in the denominator.

This process uses the difference of 2 squares. ~~and~~

Ex. $(3 - \sqrt{2})(3 + \sqrt{2}) = 9 + \cancel{3\sqrt{2}} - \cancel{3\sqrt{2}} - \sqrt{4}$

$= 9 - 2 = 7$

Simplify the expression.

17. $\frac{5}{-3 - 3\sqrt{3}}$

$\frac{5}{(-3 - 3\sqrt{3})} \cdot \frac{(-3 + 3\sqrt{3})}{(-3 + 3\sqrt{3})}$

$= \frac{5(-3 + 3\sqrt{3})}{(-3 - 3\sqrt{3})(-3 + 3\sqrt{3})}$

$= \frac{-15 + 15\sqrt{3}}{9 - \cancel{9\sqrt{3}} + \cancel{9\sqrt{3}} - 9 \cdot \sqrt{9}}$

$= \frac{-15 + 15\sqrt{3}}{9 - 9 \cdot 3} = \frac{-15 + 15\sqrt{3}}{9 - 27} = \frac{-15 + 15\sqrt{3}}{-18}$

$= \frac{\div 3 \; -15(1 - \sqrt{3})}{\div 3 \; -18} = \frac{5(1 - \sqrt{3})}{6}$

18. $\frac{3}{4 + 4\sqrt{5}}$

19. $\frac{4}{\sqrt{2} - 5\sqrt{3}}$

$\frac{4}{(\sqrt{2} - 5\sqrt{3})} \cdot \frac{(\sqrt{2} + 5\sqrt{3})}{(\sqrt{2} + 5\sqrt{3})}$

$= \frac{4\sqrt{2} + 20\sqrt{3}}{\sqrt{4} + \cancel{5\sqrt{6}} - \cancel{5\sqrt{6}} - 25\sqrt{9}}$

$= \frac{4\sqrt{2} + 20\sqrt{3}}{2 - 75} = \frac{4\sqrt{2} + 20\sqrt{3}}{-73}$

20. $\frac{\sqrt{5}}{\sqrt{3} + \sqrt{5}}$

21. The ratio of the length to the width of a *golden rectangle* is $\left(1 + \sqrt{5}\right) : 2$. The length of a golden rectangle is 62 meters. What is the width? Round your answer to the nearest meter.

The width of the rectangle is about 38 meters.

Radicals with the same index and radicand are called "like radicals". You can add and subtract like radicals the same way you combine like terms in a polynomial.

In Exercises 22–27, simplify the expression.

22. $3\sqrt{8} + 3\sqrt{2}$

23. $2\sqrt{18} - 2\sqrt{20} - 2\sqrt{5}$

$2\sqrt{9}\cdot\sqrt{2} - 2\cdot\sqrt{4}\cdot\sqrt{5} - 2\sqrt{5}$

$2\cdot 3\cdot\sqrt{2} - 2\cdot 2\cdot\sqrt{5} - 2\sqrt{5}$

$6\sqrt{2} - 4\sqrt{5} - 2\sqrt{5}$

$6\sqrt{2} - 6\sqrt{5}$

24. $3\sqrt{12} + 3\sqrt{18} + 2\sqrt{27}$

25. $2\sqrt{5}\left(\sqrt{6} + 2\right)$

26. $\left(\sqrt{7} - \sqrt{3}\right)\left(\sqrt{7} + \sqrt{3}\right)$

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$\sqrt{49} + \cancel{\sqrt{21}} - \cancel{\sqrt{21}} - \sqrt{9}$

$7 - 3 = 4$

27. $\sqrt[3]{2}\left(\sqrt[3]{108} - \sqrt[3]{135}\right)$