7.8 Trigonometric Equations (II)

θ is alone

and θ is restricted

Example 1: Solve the equation: $2\sin^2\theta - 3\sin\theta + 1 = 0,\ 0 \le \theta \le 2\pi$.

$x = \sin\theta$

$2x^2 - 3x + 1 = 0$

$a = 2 \quad b = -3 \quad c = 1$

$(2x-1)(x-1) = 0$ use ZPP

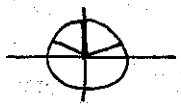
$\frac{-2}{} \cdot \frac{-1}{} = 2$

$\frac{-2}{} + \frac{-1}{} = -3$

$2x - 1 = 0$ or $x - 1 = 0$

$x = \frac{1}{2}$ or $x = 1$

	2x	-1
x	$2x^2$	-x
-1	-2x	1



$\sin\theta = \frac{1}{2}$ $\quad \sin\theta = 1$

$\theta = \frac{\pi}{6}$ or $\theta = \frac{5\pi}{6}$ $\quad \theta = \frac{\pi}{2}$

$\theta = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}\right\}$

Example 2: Solve the equation: $3\cos\theta + 3 = 2\sin^2\theta, 0 \le \theta \le 2\pi$.

problem: both sine and cosine

Solution: rewrite one function in terms of other using identities

$3\cos\theta + 3 = 2(1 - \cos^2\theta)$

$3\cos\theta + 3 = 2 - 2\cos^2\theta$

$\sin^2\theta$ ← choose this - easy to rewrite

$2\cos^2\theta + 3\cos\theta + 1 = 0$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$x = \cos\theta$

$2x^2 + 3x + 1 = 0$

$(2x+1)(x+1) = 0$

$a = 2 \quad b = 3 \quad c = 1$

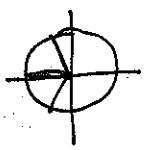
$\frac{1}{} \cdot \frac{2}{} = 2$

$\frac{1}{} + \frac{2}{} = 3$

	x	1
2x	$2x^2$	2x
1	x	1

$2x + 1 = 0$ $\quad x + 1 = 0$

$x = \frac{-1}{2}$ $\quad x = -1$



$\cos\theta = \frac{-1}{2}$ $\quad \cos\theta = -1$

$\theta = \pi$

$\theta = \frac{2\pi}{3}$

$\theta = \frac{4\pi}{3}$

$\theta = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}, \pi\right\}$

problem is the arguments don't match

Example 3: Solve the equation: $\cos(2\theta) + 3 = 5\cos\theta, 0 \le \theta \le 2\pi$

Solution:

Use $\cos(2\theta) = 2\cos^2\theta - 1$

$2\cos^2\theta - 1 + 3 = 5\cos\theta$

$2\cos^2\theta + 2 = 5\cos\theta$

$2\cos^2\theta - 5\cos\theta + 2 = 0$

$x = \cos\theta$

$2x^2 - 5x + 2 = 0$

$a = 2 \quad b = -5 \quad c = 2$

$(2x-1)(x-2) = 0$

$\frac{-4}{} \cdot \frac{-1}{} = 4$

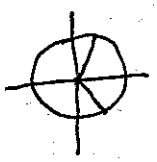
$\frac{-4}{} + \frac{-1}{} = -5$

	x	-2
2x	$2x^2$	-4x
-1	-x	2

$2x - 1 = 0$ or $x - 2 = 0$

$x = \frac{1}{2}$ $\quad x = 2$ not possible no solution

$\cos\theta = \frac{1}{2}$ $\quad \cos\theta = 2$ $\quad -1 \le 2 \le 1$ not true



$\theta = \frac{\pi}{3}$

$\theta = \frac{5\pi}{3}$

where does $\cos\theta = \frac{1}{2}$?

$\theta = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

Example 4: Solve the equation: $\cos^2\theta + \sin\theta = 2, 0 \le \theta \le 2\pi$. Check your solution(s) using your graphing calculator.

problem?: sine and cosine in problem (+ we are adding)

$1 - \sin^2\theta + \sin\theta = 2$ subtract 2

$-\sin^2\theta + \sin\theta - 1 = 0$ divide by -1

$\sin^2\theta - \sin\theta + 1 = 0$ $\quad x = \sin\theta$

$x^2 - x + 1 = 0$

$a = 1 \quad b = -1 \quad c = 1$

$\frac{}{} \cdot \frac{}{} = 1$

$\frac{}{} + \frac{}{} = -1$

not factorable

check discriminant $b^2 - 4ac$

$(-1)^2 - 4(1)(1) = -3$

negative, so no real solutions

No solution

$\theta = \{\} = \emptyset$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

To check

$Y_1 = \cos^2 x + \sin x$

$Y_2 = 2$

Example 5: Solve the equation: $\sin\theta\cos\theta = \frac{-1}{2}, 0 \le \theta \le 2\pi$

problem: sine and cosine both in problem

$2\sin\theta\cos\theta = -1$

$\sin(2\theta) = -1$

blc of argument = 2θ need the general formula

where does $\sin x = -1$? when $x = \frac{3\pi}{2}$

$2\theta = \frac{3\pi}{2} + 2k\pi$

$\theta = \frac{3\pi}{4} + k\pi$ general formula

$k = -1$

$\theta = \frac{3\pi}{4} - \frac{4\pi}{4}$

$\theta = \frac{-\pi}{4}$

X

$k = 0$

$\theta = \frac{3\pi}{4}$

✓

$k = 1$

$\theta = \frac{3\pi}{4} + \frac{4\pi}{4}$

$\theta = \frac{7\pi}{4}$

✓

$k = 2$

$\theta = \frac{3\pi}{4} + \frac{8\pi}{4}$

$\theta = \frac{11\pi}{4}$

X

$\theta = \left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

Example 6: Solve the equation: $\sin\theta + \cos\theta = 1, 0 \leq \theta \leq 2\pi$

adding is problematic

don't include 2π

Problem is $\sin\theta$ and $\cos\theta$

Square both sides but potentially this introduces extraneous solutions - so will need to check at the end.

$(\sin\theta + \cos\theta)^2 = 1^2$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 1$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$1 + 2\sin\theta\cos\theta = 1$ subtract 1

$2\sin\theta\cos\theta = 0$ use ZPP

$\sin\theta = 0$ or $\cos\theta = 0$

$\sin\theta = 0$?

$\theta = \pi \quad \theta = 0$

$\theta = \frac{\pi}{2} \quad \theta = \frac{3\pi}{2}$

$\cos\theta = 0$?

Example 8: Solve $5\sin x + x = 3$. Express the solution to two decimal places.