7.7 Solving Trigonometric Equations (I)

Example 1: Determine whether $\theta = \frac{\pi}{4}$ is a solution of the equation $2\sin\theta - 1 = 0$.
Is $\theta = \frac{\pi}{6}$ a solution?

a) $2\sin\frac{\pi}{4} - 1 \stackrel{?}{=} 0$

$2\left(\frac{\sqrt{2}}{2}\right) - 1 \stackrel{?}{=} 0$

$\sqrt{2} - 1 \stackrel{?}{\neq} 0$

False

$\frac{\pi}{4}$ is not a solution.

b) $2\sin\frac{\pi}{6} - 1 \stackrel{?}{=} 0$

$2\left(\frac{1}{2}\right) - 1 \stackrel{?}{=} 0$

$1 - 1 \stackrel{?}{=} 0$

$0 = 0$ ✓

$\frac{\pi}{6}$ is a solution

we know $\frac{5\pi}{6}$ is also a solution

Also $-\frac{7\pi}{6}$ and $-\frac{11\pi}{6}$ so is $\frac{13\pi}{6}$ and $\frac{17\pi}{6}$

There are an infinite # of solutions.

Unless the domain of a variable is restricted,
you need to find ALL of the solutions of a trigonometric equation.

1. Determine the period of the trig. function.
2. Find all solutions in the period
3. To these solutions, add multiples of the period. ← general formula

Example 2: Solve the equation: $\cos\theta = \frac{1}{2}$. Give a general formula for all the solutions.
List eight of the solutions.

$\cos\theta = \frac{1}{2}$

Period of $\cos\theta$ is 2π.

What are solutions within that period?

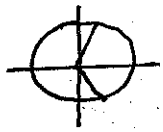
2 solutions w/in period

$\theta = \frac{\pi}{3}$

$\theta = \frac{5\pi}{3}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\cos\frac{5\pi}{3} = \frac{1}{2}$



$\left\{\theta \mid \theta = \frac{\pi}{3} + 2k\pi, \ \theta = \frac{5\pi}{3} + 2k\pi\right\}$

↳ k is an integer (so adding integer multiples of the period)

Eight solutions

$\theta = \left\{\underbrace{\frac{\pi}{3}}_{k=0}, \frac{5\pi}{3}, \underbrace{-\frac{5\pi}{3}}_{k=-1}, -\frac{\pi}{3}, \underbrace{\frac{7\pi}{3}}_{k=1}, \frac{11\pi}{3}, \underbrace{\frac{13\pi}{3}}_{k=2}, \frac{17\pi}{3}\right\}$

$k = -1$

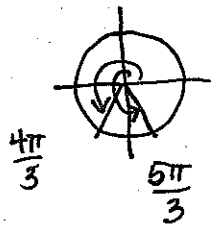
$\theta = \frac{\pi}{3} + 2(-1)\pi$

$\theta = \frac{\pi}{3} - 2\pi = \frac{\pi}{3} - \frac{6\pi}{3}$

$\theta = \frac{5\pi}{3} - \frac{6\pi}{3} = \frac{-\pi}{3}$

Example 3: Solve the equation: $2\sin\theta + \sqrt{3} = 0,\ 0 \le \theta \le 2\pi$

(θ is alone; variable is restricted)



$2\sin\theta = -\sqrt{3}$

$\sin\theta = \frac{-\sqrt{3}}{2}$

true for $\theta = \frac{4\pi}{3}$ and $\theta = \frac{5\pi}{3}$

$\theta = \left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

Example 4: Solve the equation: $\sin(2\theta) = \frac{1}{2},\ 0 \le \theta \le 2\pi$

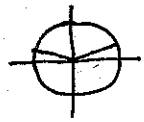
(restricted; not alone – use general formula to find all solutions in domain)

$\frac{2\theta}{2} = \frac{\pi}{6}\cdot\frac{1}{2} + \frac{2k\pi}{2}$ $\qquad$ $\frac{2\theta}{2} = \frac{5\pi}{6}\cdot\frac{1}{2} + \frac{2k\pi}{2}$

use k to find boundaries of solution set

$\theta = \frac{\pi}{12} + k\pi$ $\qquad$ $\theta = \frac{5\pi}{12} + k\pi$

$\theta = \frac{\pi}{12} + \frac{12k\pi}{12}$ $\qquad$ $\theta = \frac{5\pi}{12} + \frac{12k\pi}{12}$



T of $\sin\theta = 2\pi$

What angle gives a sine of $\frac{1}{2}$?

$\sin\frac{\pi}{6} = \frac{1}{2}$

$\sin\frac{5\pi}{6} = \frac{1}{2}$

$k=-1$

$\theta = \frac{\pi}{12} - \frac{12\pi}{12} = \frac{-11\pi}{12}$ ✗ $\qquad$ $\theta = \frac{5\pi}{12} - \frac{12\pi}{12} = \frac{-7\pi}{12}$ ✗

$k=0$

$\theta = \frac{\pi}{12}$ ✓ $\qquad$ $\theta = \frac{5\pi}{12}$ ✓

$k=1$

$\theta = \frac{\pi}{12} + \frac{12\pi}{12} = \frac{13\pi}{12}$ ✓ $\qquad$ $\theta = \frac{5\pi}{12} + \frac{12\pi}{12} = \frac{17\pi}{12}$ ✓

$k=2$

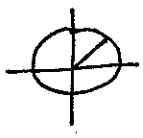
$\theta = \frac{\pi}{12} + \frac{24\pi}{12} = \frac{25\pi}{12}$ ✗

$\theta = \left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}\right\}$

Example 5: Solve the equation: $\tan\left(\theta - \frac{\pi}{2}\right) = 1,\ 0 \le \theta \le 2\pi$

θ not alone ← ← restricted

T of tangent is π



Where is $\tan x = 1$?

@ $x = \frac{\pi}{4}$

$\theta - \frac{\pi}{2} = \frac{\pi}{4} + k\pi$

$+\frac{2\pi}{4} \quad +\frac{2\pi}{4}$

$\theta = \frac{3\pi}{4} + k\pi$

$k = -1$

$\theta = \frac{3\pi}{4} - \frac{4\pi}{4} = -\frac{\pi}{4}$ X

$k = 0$

$\theta = \frac{3\pi}{4}$ ✓

$k = 1$

$\theta = \frac{3\pi}{4} + \frac{4\pi}{4} = \frac{7\pi}{4}$ ✓

$k = 2$ $\theta = \frac{3\pi}{4} + \frac{8\pi}{4} = \frac{11\pi}{4}$ X

$\theta = \left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

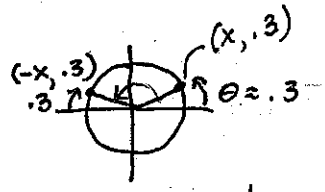
Example 6: Use a calculator to solve the equation $\sin(\theta) = 0.3,\ 0 \le \theta \le 2\pi$. Express any solutions in radians, rounded to two decimal places.

$\sin\theta = .3$

$\theta = \sin^{-1}(.3)$

$\theta \approx .30$

$\theta \approx \{.30, 2.84\}$



to find 2nd angle

$\pi - .3 \approx 2.84$