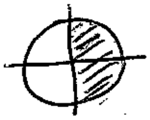
7.2 The Inverse Trigonometric Functions (continued)

Finding the exact value of expressions involving inverse trigonometric functions...

Example 1: Find the exact value of: $\sin\left(\tan^{-1}\frac{1}{2}\right) = \sin\theta = \frac{y}{r}$



$\tan^{-1}\frac{1}{2} = \theta$

means $\tan\theta = \frac{1}{2}$

must be in QI b/c $\tan\theta > 0$

$x^2+y^2=r^2 \quad \tan\theta = \frac{y}{x} = \frac{1}{2}$

$2^2+1^2=r^2 \quad y=1 \quad x=2$

$4+1=r^2$

$5=r^2 = \sqrt{5}$

$= \frac{1}{\sqrt{5}}$

$= \boxed{\frac{\sqrt{5}}{5} = \sin\left(\tan^{-1}\frac{1}{2}\right)}$

Example 2: Find the exact value of: $\cos\left[\sin^{-1}\left(\frac{-1}{3}\right)\right] = \cos\theta = \frac{x}{r}$

$\sin^{-1}\left(\frac{-1}{3}\right) = \theta$

$\sin\theta = \frac{-1}{3}$

In QIV b/c $\sin\theta < 0$

$\sin\theta = \frac{-1}{3} = \frac{y}{r}$

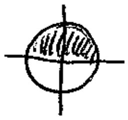
$y=-1 \quad r=3$

$x^2+(-1)^2 = 3^2$

$x^2+1=9 \quad x^2=8=2\sqrt{2}$

$= \boxed{\cos\left(\sin^{-1}\left(\frac{-1}{3}\right)\right) = \frac{2\sqrt{2}}{3}}$

Example 1: Find the exact value of: $\tan\left[\cos^{-1}\left(\frac{-1}{3}\right)\right] = \tan\theta$



$\cos^{-1}\left(-\frac{1}{3}\right) = \theta$

$\cos\theta = \frac{-1}{3} = \frac{x}{r}$

$x=-1$

$r=3$

In QII

$x^2+y^2=r^2$

$(-1)^2+y^2=3^2$

$1+y^2=9$

$y^2=8$

$y=2\sqrt{2}$

$\tan\theta = \frac{y}{x}$

$= \frac{2\sqrt{2}}{-1}$

$\boxed{\tan\left[\cos^{-1}\left(-\frac{1}{3}\right)\right] = -2\sqrt{2}}$

Example 4: Find the exact value of: $\csc^{-1} 2 = \theta$

means $\csc\theta = 2$ $\longrightarrow$ $\csc\theta = \frac{1}{\sin\theta}$

means $\sin\theta = \frac{1}{2}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$\csc^{-1} 2 = \frac{\pi}{6}$

Example 5: Use a calculator to approximate each expression in radians rounded to two decimal places.

OK on calc b/c domains the same

$0 < x < \pi$

b) $\sec^{-1} 3 = \theta$

means $\sec\theta = 3$

$0 \le x \le \pi$ $\quad \theta \ne \frac{\pi}{2}$

means $\cos\theta = \frac{1}{3}$

$\cos^{-1}\left(\frac{1}{3}\right) = \theta$

$\cos^{-1}\left(\frac{1}{3}\right) \approx 1.23$

$\boxed{\sec^{-1}(3) \approx 1.23}$

b) $\csc^{-1}(-4) = \theta$

means

$\csc\theta = -4$

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ $\quad \theta \ne 0$

can use $\sin^{-1}x$

$\sin\theta = \frac{-1}{4}$

$\sin^{-1}\left(\frac{-1}{4}\right) \approx -.25$

$\boxed{\csc^{-1}(-4) \approx -.25}$

c) $\cot^{-1}\frac{1}{2} = \theta$

means $\cot\theta = \frac{1}{2}$

can't use $\tan^{-1}x$ b/c has different domain

$\cot\theta = \frac{1}{2} = \frac{x}{y}$

will need to use $\cos^{-1}x$ b/c has domain

Need to $\cos\theta$

Know above + in Q1

$x = 1 \quad y = 2$

$1^2 + 2^2 = r^2$

$5 = r^2 \quad \sqrt{5} = r$

$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{5}}$

$\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = \theta$

≈ 1.11

$\cot^{-1}\left(\frac{1}{2}\right) \approx 1.11$

d) $\cot^{-1}(-2) = \theta \approx 2.68$

$\cot\theta = -2$

$\cot\theta = \frac{x}{y} = \frac{-2}{1}$

must be in QII

$(-2)^2 + (1)^2 = r^2$

$5 = r^2$

$\sqrt{5} = r$

$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{5}}$

$\cos^{-1}\left(\frac{-2}{\sqrt{5}}\right) = \theta$

$\theta \approx 2.68$

Example 6: Write $\sin(\tan^{-1} u)$ as an algebraic expression containing u.

no trig functions

$\sin(\tan^{-1}u) = \sin\theta$

$\sin\theta = \sin\theta \cdot \frac{\cos\theta}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta} \cdot \cos\theta$

$= \tan\theta \cdot \cos\theta$

$= \tan\theta \cdot \frac{1}{\sec\theta}$

$= \frac{\tan\theta}{\sqrt{\tan^2\theta + 1}}$

$= \frac{u}{\sqrt{u^2+1}}$

$\tan^{-1}u = \theta$

$\tan\theta = u$

D: $\tan^{-1}x$

intermediate step – write everything in terms of $\tan\theta$

$\pm\sqrt{\tan^2\theta + 1} = \sec\theta$

which root?

take + root b/c $\sec\theta > 0$ in the right hemisphere.

$\sec\theta = \sqrt{\tan^2\theta + 1}$

Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

$\tan\theta = \frac{1}{\cot\theta}$

$\cos\theta = \frac{1}{\sec\theta}$