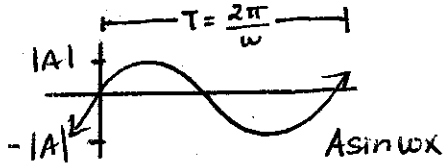
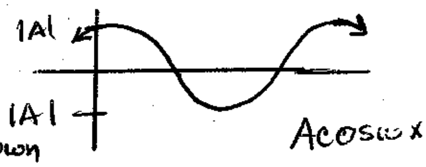


if have B
$A\sin\omega x + B$
graph moves up/down



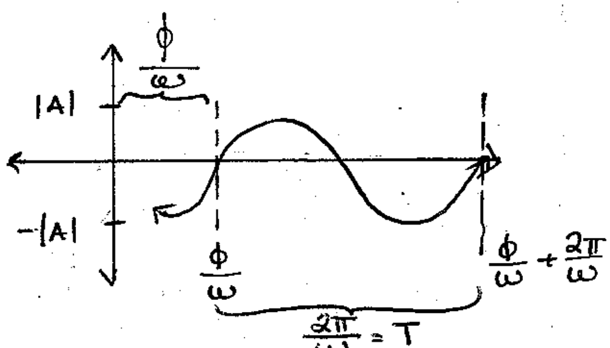
6.6 Phase Shift; Sinusoidal Curve Fitting

Phase shift notes: — horizontal shift

vertical shift - not in picture

$$A\sin(\omega x - \phi) + B = A\sin\left[\omega\left(x - \frac{\phi}{\omega}\right)\right] + B$$

phase shift: $\frac{\phi}{\omega}$



Amplitude $|A|$ $\quad \omega > 0$ always!

Period $T = \frac{2\pi}{\omega}$ $\quad$ if $\frac{\phi}{\omega} > 0$ shift right

Phase shift $= \frac{\phi}{\omega}$ $\quad$ if $\frac{\phi}{\omega} < 0$ shift left

Period begins @ $\frac{\phi}{\omega}$ and ends $\frac{\phi}{\omega} + \frac{2\pi}{\omega}$

Example 1: Find the amplitude, period and phase shift of $y = 3\sin(2x - \pi)$ and graph the function.

Use key points:

$y = 3\sin\left[2\left(x - \frac{\pi}{2}\right)\right]$

$|A| = |3| = 3$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

$\frac{\phi}{\omega} = \frac{\pi}{2}$

$A = 3$

$\omega = 2$

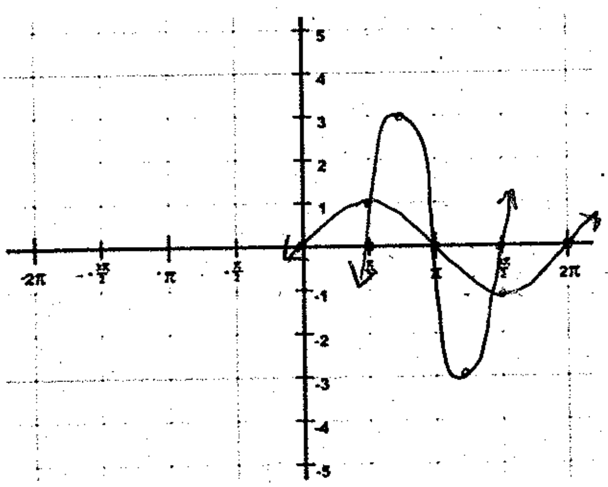
$\phi = \pi$

Divide T by 4 to get 4 subintervals.

$\frac{\pi}{4}$ $\quad \left[\frac{\pi}{2}, \frac{3\pi}{4}\right]\left[\frac{3\pi}{4}, \pi\right]\left[\pi, \frac{5\pi}{4}\right]\left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$

$\left(\frac{\pi}{2}, 0\right)\left(\frac{3\pi}{4}, 3\right)\left(\pi, 0\right)\left(\frac{5\pi}{4}, -3\right)\left(\frac{3\pi}{2}, 0\right)$

Multiply original y's by 3.



D: $\mathbb{R}$

R: $\{y \mid -3 \le y \le 3\}$

$y = 2\cos\left[4\left(x - -\frac{3\pi}{4}\right)\right] + 1$

Example 2: Find the amplitude, period and phase shift of $y = 2\cos(4x + 3\pi) + 1$ and graph the function.

$A = 2 \quad \omega = 4 \quad \phi = -3\pi \quad T: \frac{2\pi}{4} = \frac{\pi}{2} \quad \frac{\phi}{\omega} = \frac{-3\pi}{4} \quad B = 1$

Key points:

subinterval $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{8}$ $\quad \left[-\frac{3\pi}{4}, -\frac{5\pi}{8}\right]\left[-\frac{5\pi}{8}, -\frac{\pi}{2}\right]\left[-\frac{\pi}{2}, -\frac{3\pi}{8}\right]\left[-\frac{3\pi}{8}, -\frac{\pi}{4}\right]$

mult. y by 2 Key points w/o shift up $\left(-\frac{3\pi}{4}, 2\right)\left(-\frac{5\pi}{8}, 0\right)\left(-\frac{\pi}{2}, -2\right)\left(-\frac{3\pi}{8}, 0\right)\left(-\frac{\pi}{4}, 2\right)$

shift up 1 $\left(-\frac{3\pi}{4}, 3\right)\left(-\frac{5\pi}{8}, 1\right)\left(-\frac{\pi}{2}, -1\right)\left(-\frac{3\pi}{8}, 1\right)\left(-\frac{\pi}{4}, 3\right)$

Transformations:

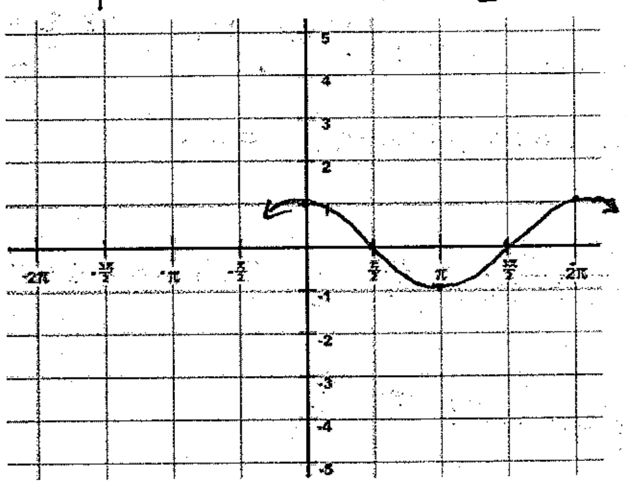
x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

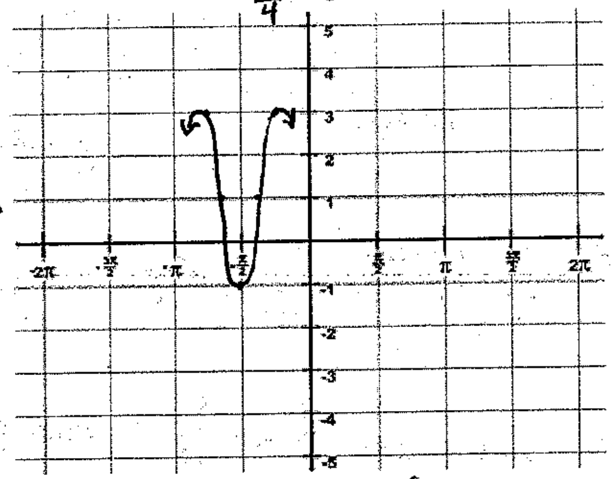
v. stretch by 2 →
h. compression by 4

x	y
0	2
$\frac{\pi}{8}$	0
$\frac{\pi}{4}$	-2
$\frac{3\pi}{8}$	0
$\frac{\pi}{2}$	2

v. shift up 1 →
h. shift left $\frac{3\pi}{4}$

x	y
$-\frac{3\pi}{4}$	3
$-\frac{5\pi}{8}$	1
$-\frac{\pi}{2}$	-1
$-\frac{3\pi}{8}$	1
$-\frac{\pi}{4}$	3





D: $\mathbb{R}$ $\quad$ R: $\{y \mid -1 \le y \le 3\}$

SUMMARY Steps for Graphing Sinusoidal Functions $y = A\sin(\omega x - \phi) + B$ or $y = A\cos(\omega x - \phi) + B$

STEP 1: Determine the amplitude $|A|$ and period $T = \frac{2\pi}{\omega}$.

STEP 2: Determine the starting point of one cycle of the graph, $\frac{\phi}{\omega}$. Determine the ending point of one cycle of the graph, $\frac{\phi}{\omega} + \frac{2\pi}{\omega}$. Divide the interval $\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + \frac{2\pi}{\omega}\right]$ into four subintervals, each of length $\frac{2\pi}{\omega} \div 4$.

STEP 3: Use the endpoints of the subintervals to find the five key points on the graph.

STEP 4: Plot the five key points with a sinusoidal graph to obtain one cycle of the graph. Extend the graph in each direction to make it complete.

STEP 5: If $B \neq 0$, apply a vertical shift.

Build Sinusoidal Models from Data

Example 3: Find a sine function that models the data in Table 11.

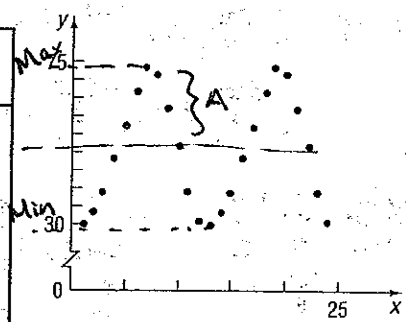
Table 11

← Denver

Month, x	Average Monthly Temperature, °F
January, 1	29.7
February, 2	33.4
March, 3	39.0
April, 4	48.2
May, 5	57.2
June, 6	66.9
July, 7	73.5
August, 8	71.4
September, 9	62.3
October, 10	51.4
November, 11	39.0
December, 12	31.0

Source: U.S. National Oceanic and Atmospheric Administration   NOAA

Figure 83



$y = A\sin(\omega x - \phi) + B$

$y = A\sin\left[\omega\left(x - \frac{\phi}{\omega}\right)\right] + B$

Find $A, B, \omega, \phi \longrightarrow \frac{\phi}{\omega}$ from data

1. Determine A: $\frac{\text{Max} - \text{Min}}{2} = \frac{73.5 - 29.7}{2} = 21.9$

$y = 21.9\sin(\omega x - \phi) + B$

2. Determine B: $\frac{\text{Max} + \text{Min}}{2} = \frac{73.5 + 29.7}{2} = 51.6$

$y = 21.9\sin(\omega x - \phi) + 51.6$

3. Determine ω
$T = 12$ $12 = \frac{2\pi}{\omega}$ $\omega = \frac{2\pi}{12} = \frac{\pi}{6}$ $y = 21.9\sin\left(\frac{\pi}{6}x - \phi\right) + 51.6$

4. Determine ϕ or $\frac{\phi}{\omega}$ Find phase shift 1* $\left(\frac{\phi}{\omega}\right)$

$T = 12$

subinterval length $\frac{12}{4} = 3$ $[0,3][3,6][6,9][9,12]$

max would occur when $x = 3$ if no phase shift.

$(0, \;)(3, \;)\ldots$ max

Max actually occurs when $x = 7$

$7 - 3 = \boxed{4 = \frac{\phi}{\omega}}$

$y = 21.9\sin\left[\frac{\pi}{6}(x - 4)\right] + 51.6$

$\phi = \frac{\phi}{\omega} \cdot \omega$

$= 4 \cdot \frac{\pi}{6} = \frac{4\pi}{6} = \boxed{\frac{2\pi}{3} = \phi}$

$y = 21.9\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 51.6$

June 21 Dec 21

Example 4: According to the *Old Farmer's Almanac*, the number of hours of sunlight in Boston on the summer solstice is 15.30 and the number of hours of sunlight on the winter solstice is 9.08.

$y = A\sin\left[\omega\left(x - \frac{\phi}{\omega}\right)\right] + B$

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \varphi) + B$ that models the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a)

a) Find A: $\frac{\text{Max} - \text{Min}}{2} = \frac{15.3 - 9.08}{2} = \boxed{A\ 3.11}$ $y = 3.11\sin x$

Find B: $\frac{\text{Max} + \text{Min}}{2} = \frac{15.3 + 9.08}{2} = \boxed{B\ 12.19}$ $y = 3.11\sin x + 12.19$

Find ω: $T = 365$ $\omega = \frac{2\pi}{T} = \boxed{\frac{2\pi}{365} = \omega}$ $y = 3.11\sin\left(\frac{2\pi}{365}x\right) + 12.19$

Find $\phi / \frac{\phi}{\omega}$: $\frac{T}{4} = \frac{365}{4} = 91.25$ phase shift

[0, 91.25][91.25, 182.5] . . .

max occurs here with no phase shift

Jan 31
Feb 28
March 31
April 30
May 31
June 21
172

$172 - 91.25 = \boxed{\frac{\phi}{\omega} = 80.75}$

$$\boxed{y = 3.11\sin\left[\frac{2\pi}{365}(x - 80.75)\right] + 12.19}$$

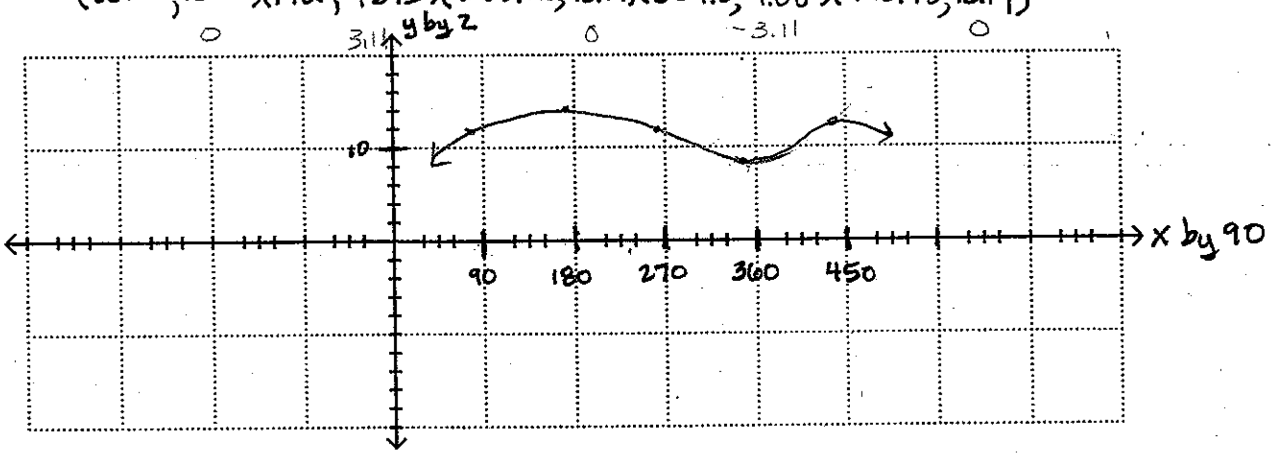
$\phi = 80.75 \cdot \frac{2\pi}{365} = \frac{161.5\pi}{365} \cdot \frac{2}{2} = \boxed{\frac{323\pi}{730} = \phi}$

$$\boxed{y = 3.11\sin\left(\frac{2\pi}{365}x - \frac{323\pi}{730}\right) + 12.19}$$

b) $y = 3.11\sin\left[\frac{2\pi}{365}(91 - 80.75)\right] + 12.19 \approx 12.74$

Our model predicts that Boston will have about 12 hours and 44 minutes of sunlight on April 1st.

(80.75, 12.19)(172, 15.3)(263.25, 12.19)(354.5, 9.08)(445.75, 12.19)



Example 5: Use a graphing calculator to find the sine function of best fit that models the data in question 3. Graph this function with the scatter plot of the data.

$$y = 21.15 \sin(.55x - 2.35) + 51.19$$

(Be sure your calculator is in radian mode!)