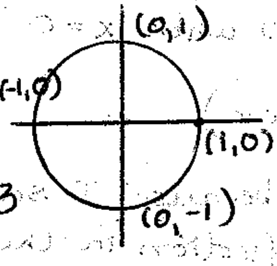
6.5 Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions

The Tangent Function:

$x = \theta$	$\frac{\sin\theta}{\cos\theta}$	$\tan\theta$
$-\frac{\pi}{2}$	$\frac{-1}{0}$	v. asymptote undefined
$-\frac{\pi}{3}$	$-\frac{\sqrt{3}}{2} / \frac{1}{2}$	$-\sqrt{3} \approx -1.73$
$-\frac{\pi}{4}$	$-\frac{\sqrt{2}}{2} / \frac{\sqrt{2}}{2}$	-1
$-\frac{\pi}{6}$	$\frac{-1}{2} / \frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3} \approx -.58$
0	$\frac{0}{1}$	0
$\frac{\pi}{6}$	$\frac{1}{2} / \frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3} \approx .58$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} / \frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2} / \frac{1}{2}$	$\sqrt{3} \approx 1.73$
$\frac{\pi}{2}$	$\frac{1}{0}$	ud

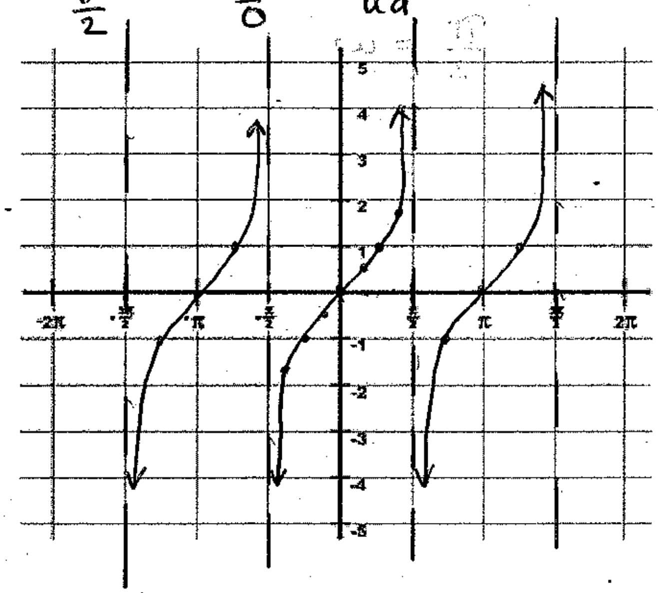


$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

period $= \pi = 180°$

$\tan\theta$ is undefined when $\cos\theta = 0$

@ odd integer multiples of $\frac{\pi}{2}$



6. x-intercepts at integer multiples of π

y-intercept $(0,0)$

(No amplitude)

Properties of the Tangent Function:

1. D: $\mathbb{R}$ except odd integer multiples of $\frac{\pi}{2}$.

   D: $\left\{x \mid x \neq \frac{k\pi}{2}, k \text{ is an odd integer}\right\}$

   R: $\mathbb{R}$
2. Periodic with a $T = \pi = 180°$
3. Vertical asymptotes at odd integer multiples of $\frac{\pi}{2}$.

   No horizontal asymptote.
4. Tangent is odd so the graph is symmetric about the origin.
5. Increasing function b/t vertical asymptotes. Smooth + continuous b/t vertical asymptotes.

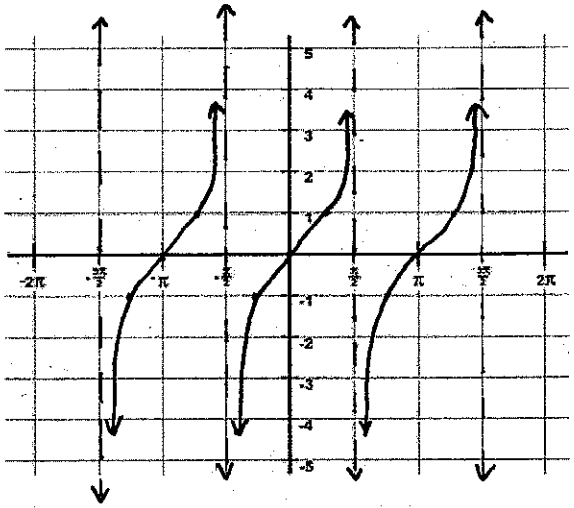
v. stretch   shift down 1

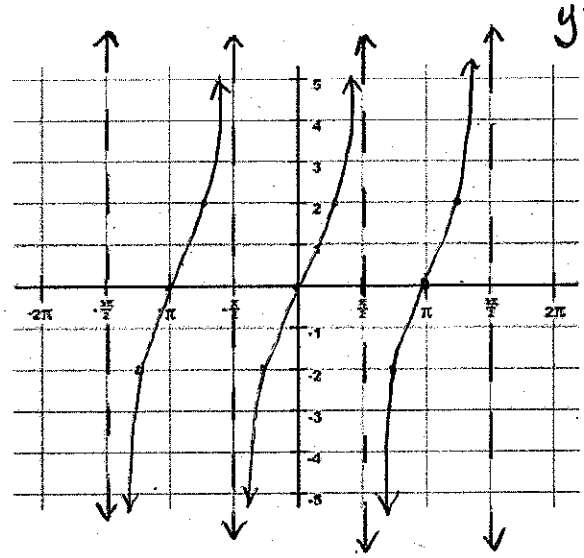
Example 1: Graph y = 2tan$x$ – 1 using transformations.

Use the graph to determine the domain and the range of y = 2tan$x$ – 1.

$y = \tan x$

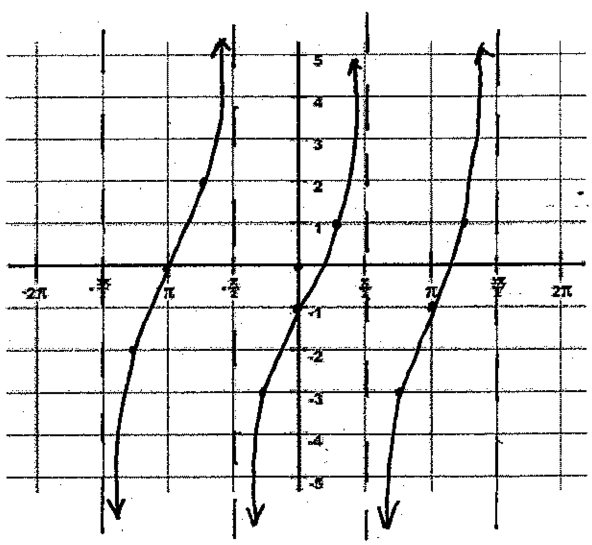
x	y
$-\frac{\pi}{2}$	ud
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	ud





$y = 2\tan x$

x	y
$-\frac{\pi}{2}$	ud
$-\frac{\pi}{4}$	-2
0	0
$\frac{\pi}{4}$	2
$\frac{\pi}{2}$	ud



$y = 2\tan x - 1$

x	y
$-\frac{\pi}{2}$	ud
$-\frac{\pi}{4}$	-3
0	-1
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	ud

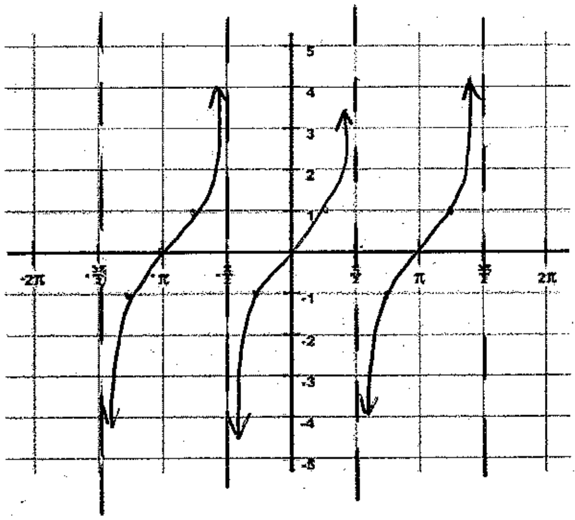
D: $\mathbb{R}$ except integer odd multiples of $\frac{\pi}{2}$

R: $\mathbb{R}$

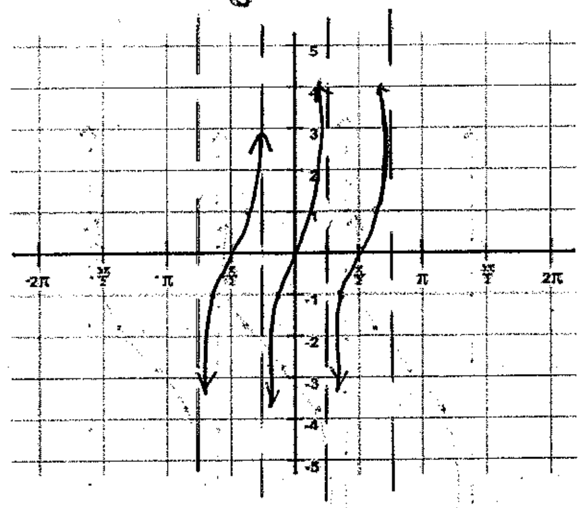
Example 2: ~~T~~ G raph y = 3tan(2*x*) using transformations. Use the graph to determine the domain and the range of y = 3tan(2*x*).

$y = \tan x$

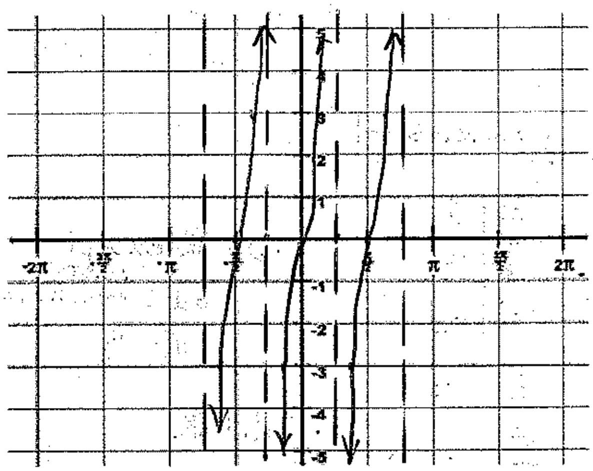
x	y
$-\frac{\pi}{2}$	ud
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	ud



$y = \tan(2x)$



x	y
$-\frac{\pi}{4}$	ud
$-\frac{\pi}{8}$	-1
0	0
$\frac{\pi}{8}$	1
$\frac{\pi}{4}$	ud



$y = 3\tan(2x)$

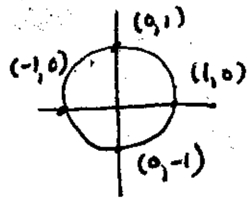
x	y
$-\frac{\pi}{4}$	ud
$-\frac{\pi}{8}$	-3
0	0
$\frac{\pi}{8}$	3
$\frac{\pi}{4}$	ud

D: $\mathbb{R}$ except odd integer multiples of $\frac{\pi}{4}$

R : $\mathbb{R}$

The Cotangent Function:

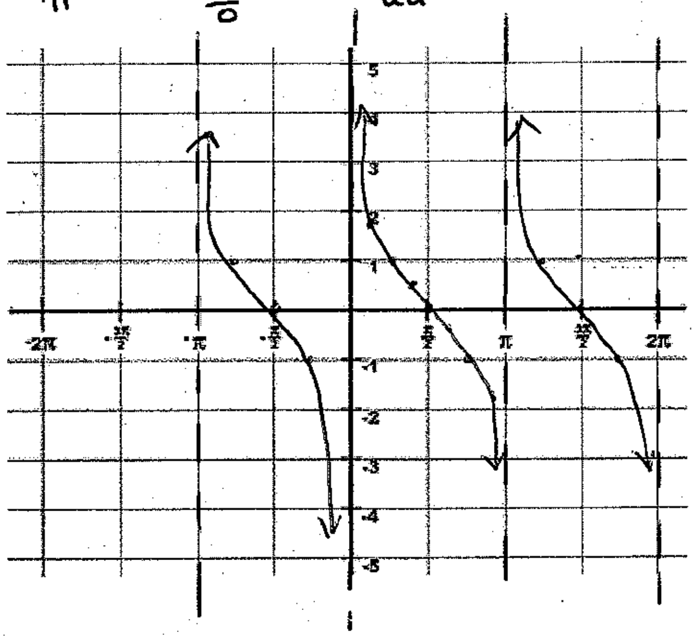
X	$\frac{\cos\theta}{\sin\theta}$	$\cot\theta$
0	$\frac{1}{0}$	ud
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}/\frac{1}{2}$	$\sqrt{3} \approx 1.73$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}/\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{1}{2}/\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3} \approx .58$
$\frac{\pi}{2}$	$\frac{0}{1}$	0
$\frac{2\pi}{3}$	$\frac{-1}{2}/\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3} \approx -.58$
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}/\frac{\sqrt{2}}{2}$	-1
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}/\frac{1}{2}$	$-\sqrt{3} \approx -1.73$
π	$\frac{-1}{0}$	ud



$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{x}{y}$$

undefined where $\sin\theta = 0$

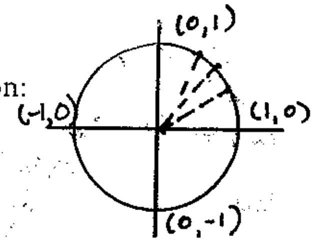
integer multiples of π



Properties of cotangent

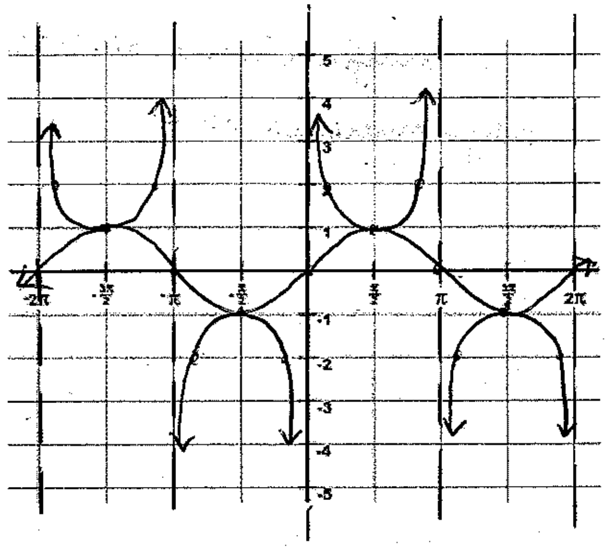
1. D: $\mathbb{R}$ except integer multiples of π
   R: $\mathbb{R}$
2. Periodic with a period of π
3. Vertical asymptotes @ integer multiples of π.
   No horizontal asymptotes.
4. Function is odd and graph is symmetric about the origin.
5. Decreasing function between asymptotes
6. Smooth + continuous between asymptotes
7. No y-intercept
   X-intercepts: @ odd integer multiples of $\frac{\pi}{2}$

The Cosecant Function and the Secant Function:

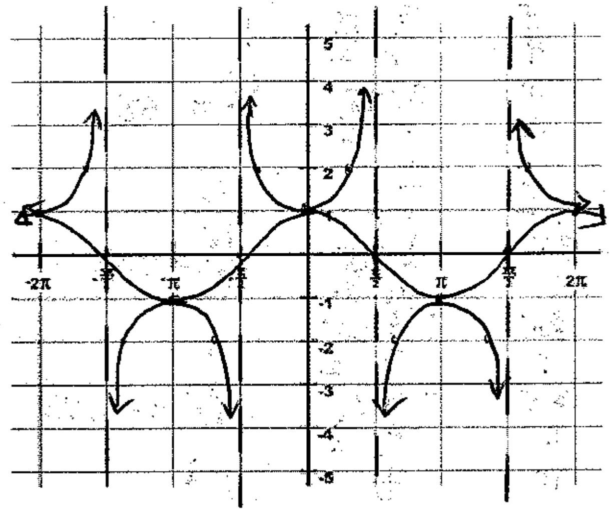


$y = \csc x = \frac{1}{\sin x}$

x	y
0	ud
$\frac{\pi}{6}$	2
$\frac{\pi}{2}$	1
$\frac{5\pi}{6}$	2
π	ud
$\frac{7\pi}{6}$	-2
$\frac{3\pi}{2}$	-1
$\frac{11\pi}{6}$	-2
2π	ud



$y = \sec x = \frac{1}{\cos x}$



x	y
0	1
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	ud
$\frac{2\pi}{3}$	-2
π	-1
$\frac{4\pi}{3}$	-2
$\frac{3\pi}{2}$	ud
$\frac{5\pi}{3}$	2
2π	1

Properties of cosecant

1) D: $\mathbb{R}$ except integer multiples of π.

   R: $\{y \mid y \le -1 \text{ or } y \ge 1\}$

2) Periodic with a period of 2π.

3) Vertical asymptotes at integer multiples of π.

   No horizontal asymptotes.

4) odd function so graph is symmetric about the origin.

5) No x-intercepts
   No y-intercepts

Properties of secant

1) D: $\mathbb{R}$ except odd integer multiples of $\frac{\pi}{2}$

   R: same

2) same

3) Vertical asymptotes at odd integer multiples of $\frac{\pi}{2}$
   No horizontal asymptotes

4) Even function so graph is symmetric about the y-axis.

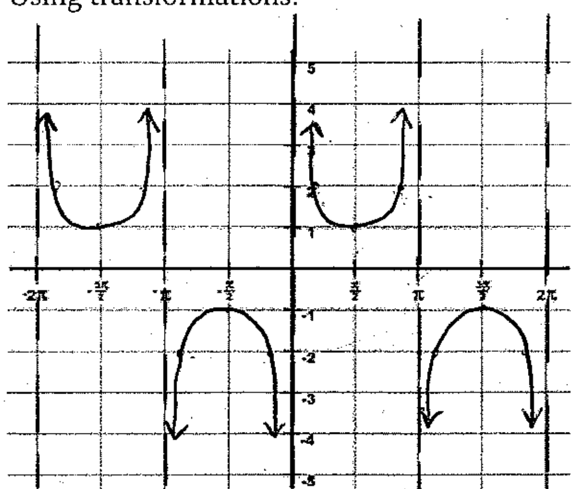
5) No x-intercepts
   y-intercept $(0,1)$

Example 3: Graph y = 2csc x – 1 using transformations or using the reciprocal function. Use the graph to determine the domain and the range of y = 2csc x – 1.

Using transformations:

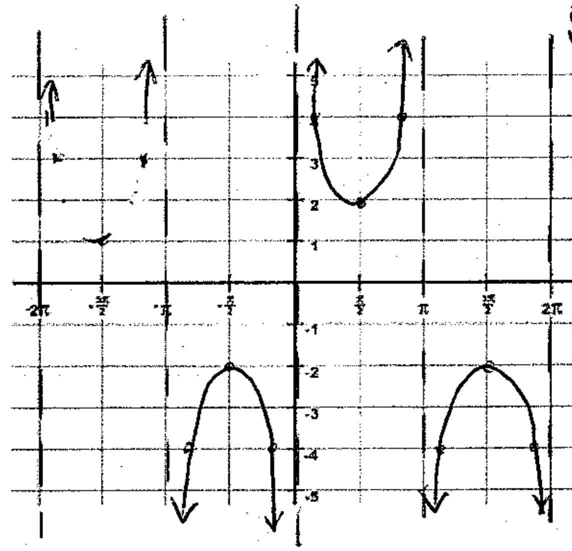
$y = \csc x$

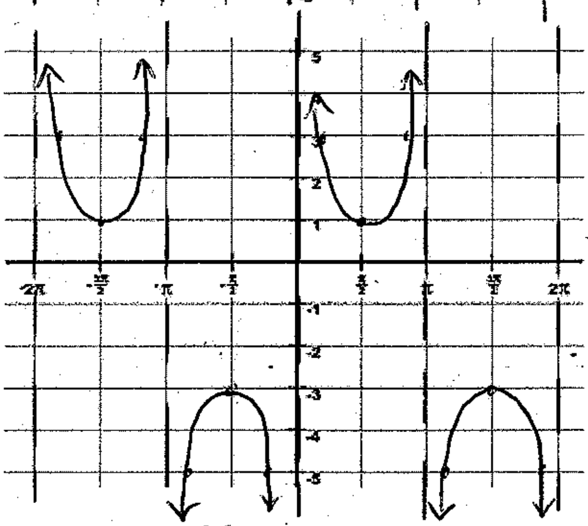
x	y
0	ud
$\frac{\pi}{2}$	1
π	ud
$\frac{3\pi}{2}$	−1
2π	ud



$y = 2\csc x$

x	y
0	ud
$\frac{\pi}{2}$	2
π	ud
$\frac{3\pi}{2}$	−2
2π	ud





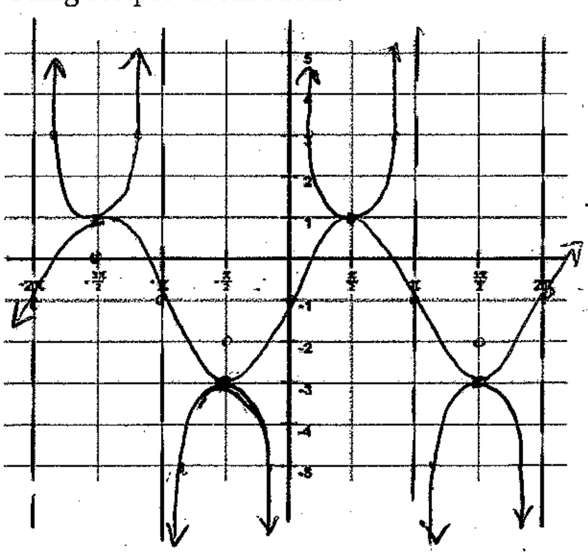
$y = 2\csc x - 1$

x	y
0	ud
$\frac{\pi}{2}$	1
π	ud
$\frac{3\pi}{2}$	−3
2π	ud

D: $\mathbb{R}$ except integer multiples of π

R: $\{y \mid y \geq 1 \text{ or } y \leq -3\}$

Using reciprocal functions:



$y = 2\csc x - 1$

reciprocal function = sine

$y = 2\sin x - 1$ ← graph this

$y = \sin x \longrightarrow y = 2\sin x \longrightarrow y = 2\sin x - 1$

x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	−1
2π	0

x	y
0	0
$\frac{\pi}{2}$	2
π	0
$\frac{3\pi}{2}$	−2
2π	0

x	y
0	−1
$\frac{\pi}{2}$	1
π	−1
$\frac{3\pi}{2}$	−3
2π	−1