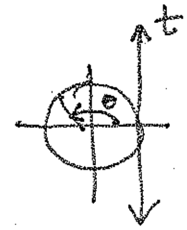


6.4 Graphs of the Sine and Cosine Functions

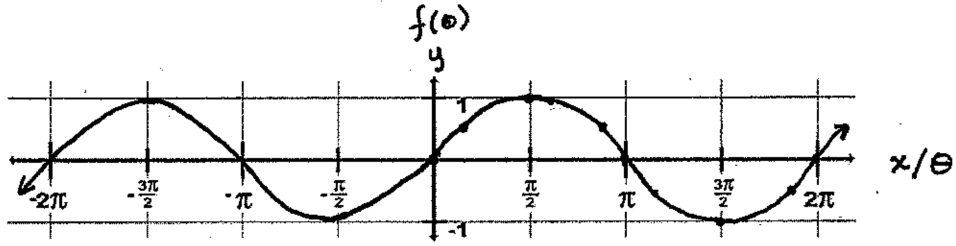
θ → f(θ)

The graph of the sine function y = sinx ← book is graphing x/y plane

radians

Key points $f(\theta) = \sin\theta$ Graph and Properties:

θ x	f(θ) y
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1
$\frac{5\pi}{6}$	$\frac{1}{2}$
π	0
$\frac{7\pi}{6}$	$-\frac{1}{2}$
$\frac{3\pi}{2}$	-1
$\frac{11\pi}{6}$	$-\frac{1}{2}$
2π	0



1. D: ℝ R: $\{y \mid -1 \le y \le 1\}$
2. Sine functions is odd so the graph is symmetric about the origin.
3. Sine function is periodic - with a period of 2π.
4. x-int. ..: $-2\pi, -\pi, 0, \pi, \ldots$ (integer multiples of π)
5. (0,0) y-int.
6. Maximum → 1
   Minimum → −1 $|A| > 1$ $0 < |A| < 1$ if A is negative
   ← vertical stretch/compression → also reflections over x-axis
7. Smooth and continuous

Graphing Functions of the Form $y = A\sin(\omega x)$ → horizontal stretch $0 < \omega < 1$ h. compression $\omega > 1$

ω always +

Example 1: Graph y = 3 sin x using transformations. Use the graph to determine the domain and the range of y = 3 sinx.

v. stretch by 3

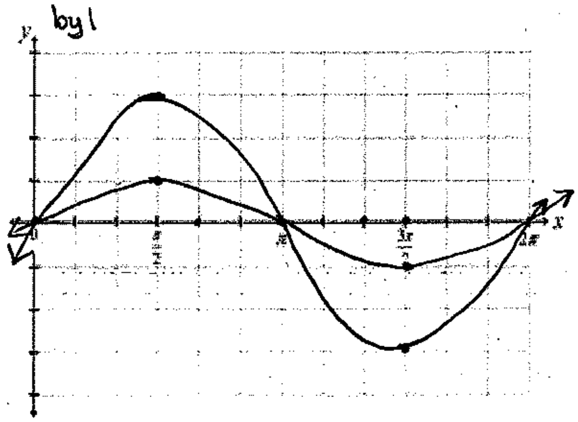
y = sinx

x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

y = 3sinx

x	y
0	0
$\frac{\pi}{2}$	3
π	0
$\frac{3\pi}{2}$	-3
2π	0

by 1



D: ℝ

R: $\{y \mid -3 \le y \le 3\}$

Example 2: Graph y = - sin(2x) using transformations. Use the graph to determine the domain and the range of y = -sin(2x).

$y = \sin x$

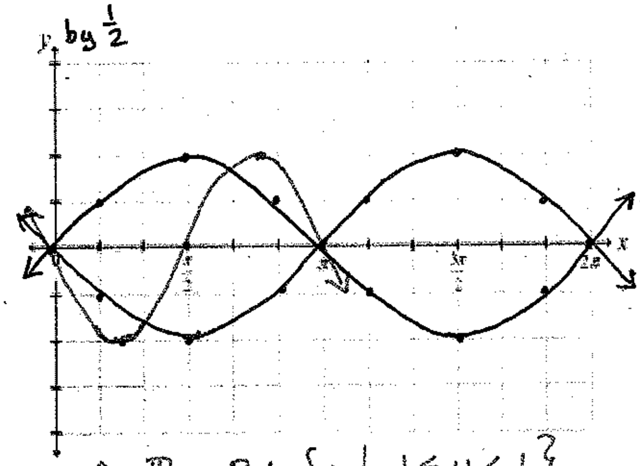
x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$y = -\sin x$

x	y
0	0
$\frac{\pi}{2}$	-1
π	0
$\frac{3\pi}{2}$	1
2π	0

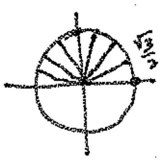
$y = -\sin(2x)$

x	y
0	0
$\frac{\pi}{4}$	-1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	1
π	0



D: $\mathbb{R}$ R: $\{y \mid -1 \le y \le 1\}$

Period: π



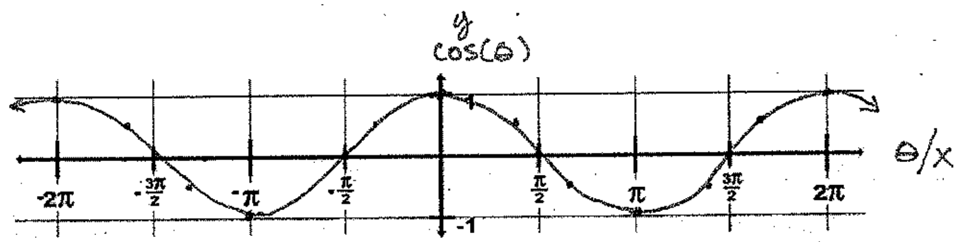
The graph of the cosine function y = cosx

Key points

θ x	$\cos(\theta)$ y
0	1
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$-\frac{1}{2}$
π	-1
$\frac{4\pi}{3}$	$-\frac{1}{2}$
$\frac{3\pi}{2}$	0
$\frac{5\pi}{3}$	$\frac{1}{2}$
2π	1

cos(0)

Graph and Properties:



1. D: $\mathbb{R}$ R: $\{y \mid -1 \le y \le 1\}$
2. Cosine function is even so the graph is symmetric about the y-axis.
3. Periodic with a period of 2π.
4. x-int: odd Integer multiples of $\frac{\pi}{2}$
   y-int: (0, 1)
5. Max: 1
   Min: -1
6. Smooth and continuous

Graphing Functions of the Form y = Acos(ωx)

Example 3: Graph y = 2 cos(3x) using transformations. Use the graph to determine the domain and the range of y = 2 cos(3x).

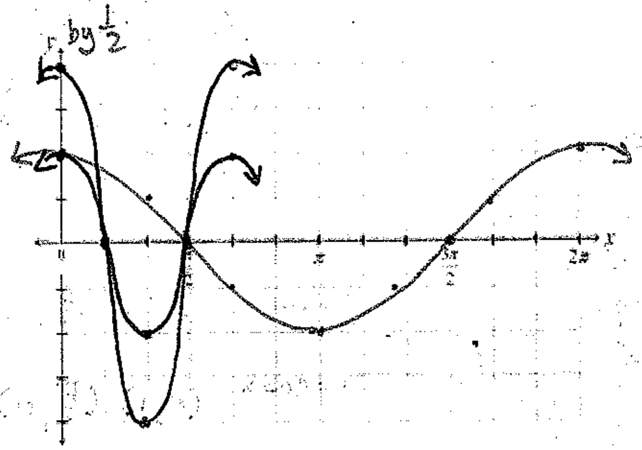
$y = \cos x \xrightarrow[\text{compression}]{h.} \cos(3x) \xrightarrow[\text{stretch}]{v.} y = 2\cos(3x)$

x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

$x \cdot \frac{1}{3}$

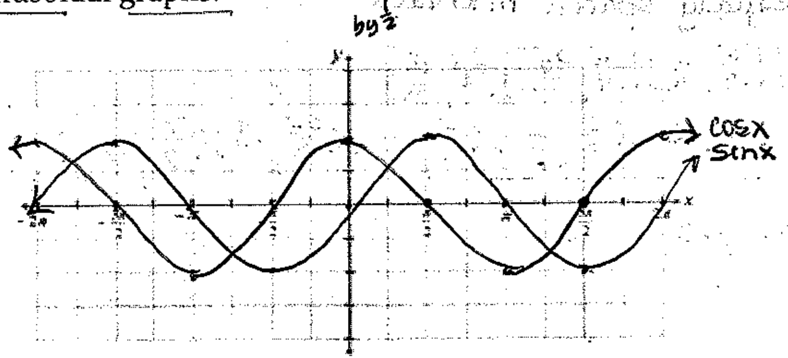
x	y
0	1
$\frac{\pi}{6}$	0
$\frac{\pi}{3}$	-1
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	1

x	y
0	2
$\frac{\pi}{6}$	0
$\frac{\pi}{3}$	-2
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	2



D: $\mathbb{R}$ R: $\{y \mid -2 \le y \le 2\}$

Sinusoidal graphs:



1. cosx
2. sinx
3. $\cos\left(x - \frac{\pi}{2}\right)$

shift right of $\frac{\pi}{2}$

$\cos\left(x - \frac{\pi}{2}\right) = \sin x$

4. $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

Both sine + cosine are sinusoidal graphs.

Use sinx for modeling.

$y = A\sin(\omega x)$ $\qquad$ $y = A\cos(\omega x)$

Amplitude = $|A|$ $\qquad$ Range: $-|A| \le y \le |A|$

Period = T $\qquad$ $T = \frac{2\pi}{\omega}$ $\qquad$ $\omega > 0$

use even/odd to make +

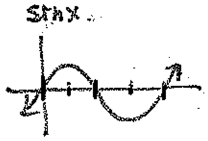
Example 4: Determine the amplitude and period of y = 5 sin(-4x).

$y = -5\sin(4x)$ pull out − b/c sine odd.

$|A| = |-5| = 5$

Period $= T = \frac{2\pi}{4} = \frac{\pi}{2}$

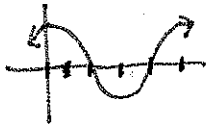
sin x



Each function has 4 intervals

$[0, \frac{\pi}{2}]\,[\frac{\pi}{2}, \pi]\,[\pi, \frac{3\pi}{2}]\,[\frac{3\pi}{2}, 2\pi]$

cos x



$y = \sin x$ $(0,0)\ (\frac{\pi}{2}, 1)\ (\pi, 0)\ (\frac{3\pi}{2}, -1)\ (2\pi, 0)$

$y = \cos x$ $(0,1)\ (\frac{\pi}{2}, 0)(\pi, -1)\ (\frac{3\pi}{2}, 0)\ (2\pi, 1)$

Example 5: Graph y = 3 sin(4x) using key points. Id A+ω

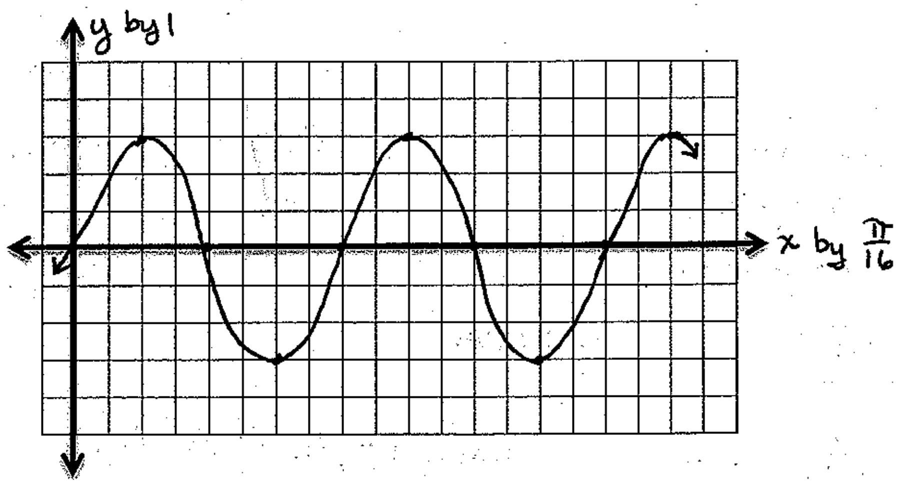
1. Determine A + T $\quad A = 3 \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$

2. Divide T into 4 equally spaced intervals

$\frac{\pi}{2} \div 4 = \frac{\pi}{8} \quad [0, \frac{\pi}{8}][\frac{\pi}{8}, \frac{\pi}{4}][\frac{\pi}{4}, \frac{3\pi}{8}][\frac{3\pi}{8}, \frac{\pi}{2}]$

3. Write key points $(0, 0)(\frac{\pi}{8}, 3)(\frac{\pi}{4}, 0)(\frac{3\pi}{8}, -3)(\frac{\pi}{2}, 0)$

Multiply "old" y's by A to get new y's. $y \cdot 3$

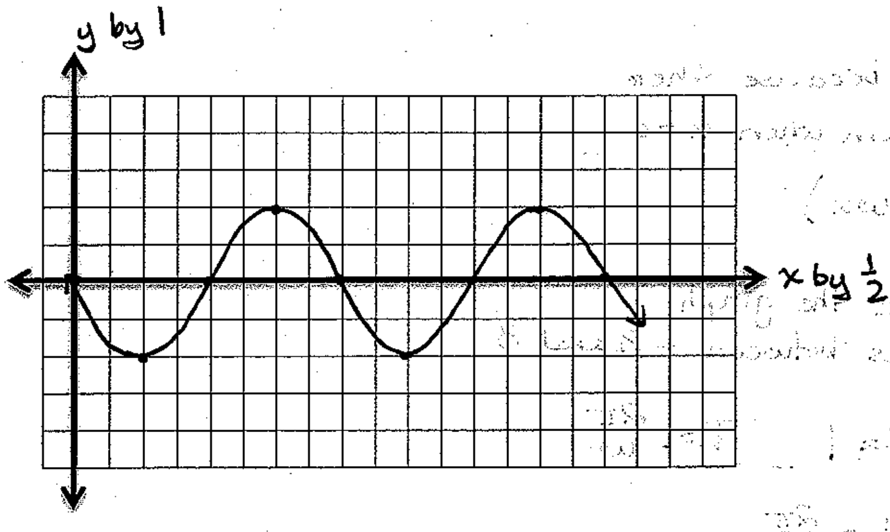


Summary: Steps for graphing using key points.

1. Determine A and T (period) $\quad T = \frac{2\pi}{\omega}$
2. Divide $\left[0, \frac{2\pi}{\omega}\right]$ into 4 subintervals of equal length.
   [ ][ ][ ][ ]
3. Use endpoints of subintervals as x-coordinates of key points. To find y-coordinates of key points, multiply "old" y's by A. Gives 5 key points.
4. Plot key points and join with curve.
5. Shift graph if necessary. (can put step 5 before step 4)

Example 6: Graph $y = 2\sin\left(-\frac{\pi}{2}x\right)$ using key points. $y = -2\sin\left(\frac{\pi}{2}x\right)$ b/c sine is odd.

1. $A = -2 \qquad T = \frac{2\pi}{\frac{\pi}{2}} = 2\pi \div \frac{\pi}{2} = \frac{2\cancel{\pi}}{1} \cdot \frac{2}{\cancel{\pi}} = 4$
   $\omega = \frac{\pi}{2}$
2. $\frac{4}{4} = 1 \qquad [0,1][1,2][2,3][3,4]$ $\qquad$ multiply $y \cdot (-2)$
3. $(0,0)\ (1,-2)\ (2,0)\ (3,2)\ (4,0)$

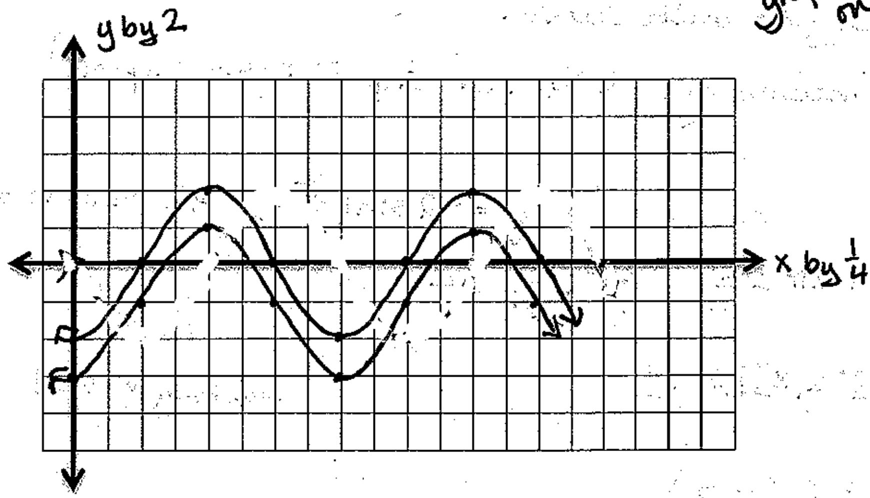


$(0,1)\ (\frac{\pi}{2},0)\ (\pi,-1)\ (\frac{3\pi}{2},0)\ (2\pi,1)$

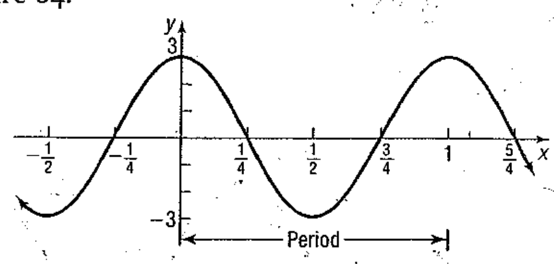
Example 7: Graph $y = -4\cos(\pi x) - 2$ using key points. Use the graph to determine the domain and the range of $y = -4\cos(\pi x) - 2$.

1. $A = -4 \quad \omega = \pi \qquad T = \frac{2\pi}{\pi} = 2$
2. $2 \div 4 = \frac{1}{2} \qquad [0, \frac{1}{2}][\frac{1}{2}, 1][1, \frac{3}{2}][\frac{3}{2}, 2] \qquad$ y's $\cdot -4$
3. $(0,-4)(\frac{1}{2},0)(1,4)(\frac{3}{2},0)(2,-4)$
4. $(0,-6)(\frac{1}{2},-2)(1,2)(\frac{3}{2},-2)(2,-6)$

graph this on test or quiz



Example 8: Finding an equation for the graph show in Figure 64.



looks like a cosine because there is a maximum when $x = 0$

$y = A\cos(\omega x)$

$A = 3$ because the graph ranges between -3 and 3

$T = 1 \qquad T = \frac{2\pi}{\omega}$

$1 = \frac{2\pi}{\omega}$

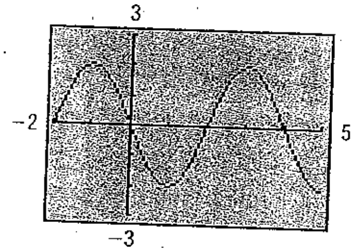
$\omega = 2\pi$

$y = 3\cos(2\pi x)$

Example 9: Find an equation for the graph shown in Figure 65.

$\Rightarrow$ sine b/c $y = 0$ when $x = 0$

Figure 65



$y = A\sin(\omega x)$

A is negative because I see a reflection in the graph

Range: $-2 \le y \le 2$

$A = -2$

$T = 4$

$\frac{2\pi}{\omega} = 4$

$2\pi = 4\omega$

$\frac{2\pi}{4} = \omega$

$\frac{\pi}{2} = \omega$

$y = -2\sin\left(\frac{\pi}{2}x\right)$