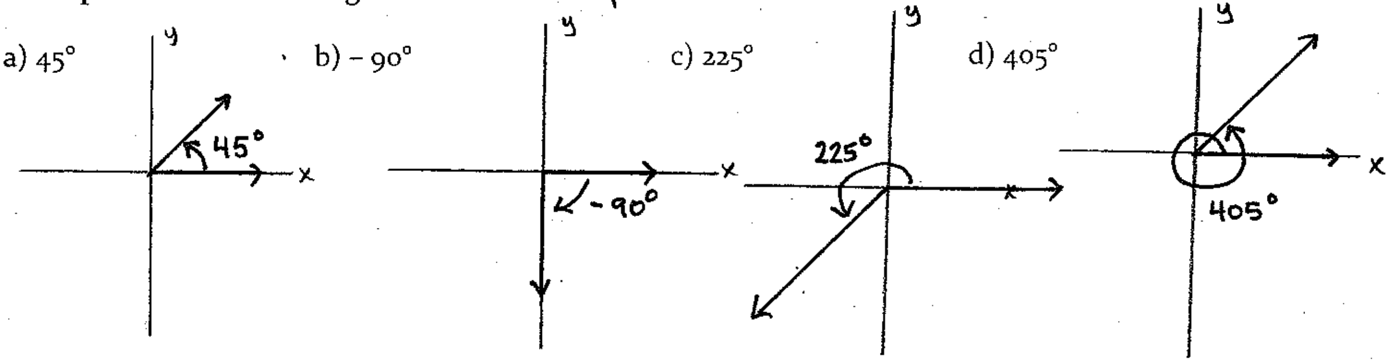
Example 1. Draw each angle. (in standard position)

a) $45°$ b) $-90°$ c) $225°$ d) $405°$



Subdivisions of degrees: - can use decimals (calculations) or minutes and seconds (navigation - tells position of ship/plane latitude/longitude)

one minute = $\frac{1}{60}$ degree   denoted one minute = $1'$

one second = $\frac{1}{60}$ minute = $\frac{1}{3600}$ degree   denoted one second = $1''$

1 counterclockwise revolution = $360°$ $1° = 60'$ $1' = 60''$

Example 2.

a) Convert $50°6'21''$ to a decimal in degrees. Round your answer to four decimal places.

$$50°6'21'' = 50° + \cancel{6'} \cdot \frac{1°}{\cancel{60'}_{10}} + 21'' \cdot \frac{1°}{3600''}$$

$$= 50° + .1° + .0058°$$

$$\approx 50.1058°$$

ON GRAPHING CALC - In degree mode?
50 2nd apps (Angle) → 1: °
6 2nd apps (Angle) → 2: '
21 Alpha + (")
then hit enter
≈ 50.1058°

b) Convert $21.256°$ to the D°M'S" form. Round the answer to the nearest second.

$$21.256° = 21° + .256° \cdot \frac{60'}{1°}$$

$$= 21° + 15.36'$$

$$= 21°15' + .36' \cdot \frac{60''}{1'}$$

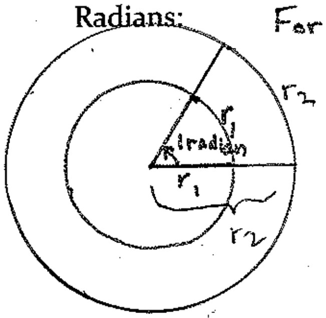
$$= 21°15' + 21.6''$$

$$\approx 21°15'22''$$

Enter
21.256°
2nd apps (Angle) → 4: DMS
then hit enter
= 21°15'21.6"

used in calculus

Radians: For a circle of radius r, the rays of a central angle with measure 1 radian, subtend an arc of length r.

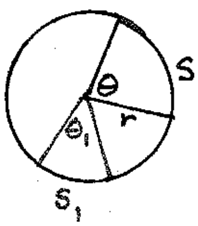


1 revolution = 360° = 2π radians

$\frac{1}{2}$ rev = 180° = π radians

typically left off

Arc Length Theorem: Consider a circle of radius r and 2 central angles (θ and θ_1 – measured in radians). Suppose that these angles subtend arcs of lengths s and s_1.



From Geo we remember

$$\frac{\theta}{\theta_1} = \frac{s}{s_1}$$

suppose that θ_1 = 1 radian then $s_1 = r$

$$\frac{\theta}{1} = \frac{s}{r}$$

$$\boxed{s = r\theta}$$

Example 3. Find the length of the arc of a circle of radius 2 meters subtended by a central angle of 0.25 radian.

$s = ?$ $\quad r = 2\text{m}$ $\quad \theta = .25$

$s = 2(.25)$

$s = .5$

The length of the arc is $\frac{1}{2}$ meter.

Converting from degrees to radians and from radians to degrees

degrees $\longrightarrow$ radians $\quad \frac{{}^\circ}{1} \cdot \frac{\pi \text{ radians}}{180^\circ}$

radians $\longrightarrow$ degrees $\quad \frac{\text{radians}}{1} \cdot \frac{180^\circ}{\pi \text{ radians}}$

Example 4. Convert each angle in degrees to radians.

a) $\cancel{60^\circ}^{1} \cdot \frac{\pi}{\cancel{180^\circ}_{3}} = \frac{\pi}{3}$ radians

b) $\cancel{150^\circ}^{5} \cdot \frac{\pi}{\cancel{180^\circ}_{6}} = \frac{5\pi}{6}$ radians

c) $\cancel{-45^\circ}^{-1} \cdot \frac{\pi}{\cancel{180^\circ}_{4}} = \frac{-\pi}{4}$ radians

d) $\cancel{90^\circ}^{1} \cdot \frac{\pi}{\cancel{180^\circ}_{2}} = \frac{\pi}{2}$ radians

e) $107^\circ \cdot \frac{\pi}{180^\circ} = \frac{107\pi}{180}$

≈ 1.868 radians

Example 5. Convert each angle in radians to degrees.

a) $\frac{\cancel{\pi}}{\cancel{6}_1}$ radian $\cdot \frac{\cancel{180^\circ}^{30}}{\cancel{\pi}} = 30^\circ$

b) $\frac{3\cancel{\pi}}{\cancel{2}_1}$ radians $\cdot \frac{\cancel{180^\circ}^{90^\circ}}{\cancel{\pi}} = 270^\circ$

c) $\frac{-3\cancel{\pi}}{\cancel{4}_1}$ radians $\cdot \frac{\cancel{180^\circ}^{45}}{\cancel{\pi}} = -135^\circ$

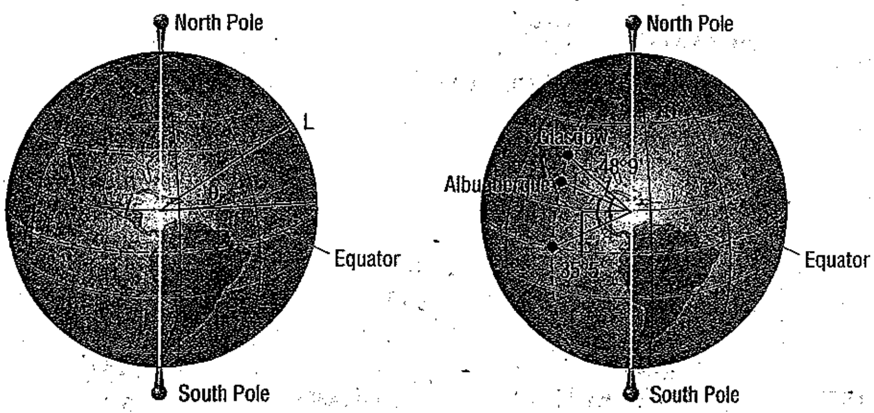
d) $\frac{7\cancel{\pi}}{\cancel{3}_1}$ radians $\cdot \frac{\cancel{180^\circ}^{60}}{\cancel{\pi}} = 420^\circ$

:) I learned something today.

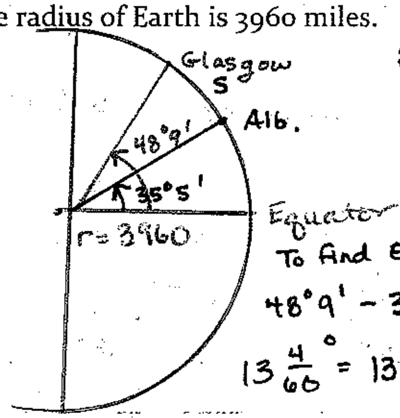
e) 3 radians $\cdot \frac{180^\circ}{\pi} = \frac{540^\circ}{\pi} \approx 171.887^\circ$

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		210°	225°	240°	270°	300°	315°	330°	360°
Radians	--------	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

Example 6. The latitude of a location L is the angle formed by a ray drawn from the center of Earth to the Equator and a ray drawn from the center of Earth to L. Glasgow, Montana, is ~~die~~ due north of Albuquerque, New Mexico. Find the distance between Glasgow (48°9' north latitude) and Albuquerque (35°5' north latitude). Assume that the radius of Earth is 3960 miles.



(a) (b)



$S = r\theta$

in radians

To find θ

$48^\circ 9' - 35^\circ 5' = 13^\circ 4'$

$13\frac{4}{60}^\circ = 13\frac{1}{15}^\circ = \frac{196^\circ}{15}$

$\frac{196^\circ}{15} \cdot \frac{\pi}{180} \approx .2281$ radians

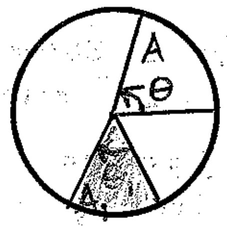
$S = (3960)(.2281)$

$S \approx 903$

round to nearest mile

Glasgow is about 903 miles north of Albuquerque.

Area of a Sector of a Circle Theorem: Consider a circle with radius r and 2 central angles (θ and θ_1 measured in radians),



From Geo we remember

$$\frac{\theta}{\theta_1} = \frac{A}{A_1}$$

Suppose $\theta_1 = 2\pi$ then $A_1 = \pi r^2$

substitute

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

$$\frac{\theta \cancel{\pi} r^2}{2\cancel{\pi}} = \frac{2\pi A}{2\pi}$$

$$\boxed{A = \frac{1}{2} r^2 \theta}$$

Example 7. Find the area of the sector of a circle of radius 2 feet formed by an angle of 30°. Round your answer to two decimal places. $A = ?$ $r = 2\text{ ft.}$ $\theta = \cancel{30°} \cdot \frac{\pi}{\cancel{180°}_6} = \frac{\pi}{6}$

$$A = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2}(2)^2\left(\frac{\pi}{6}\right)$$

$$A = \frac{2\pi}{6} = \frac{\pi}{3} \approx 1.05\text{ ft}^2$$

The area of the sector is about 1.05 ft^2.

Definition – Linear speed: – the type you are most familiar with

old formula

$d = rt$ $\quad r = \frac{d}{t}$

$$\boxed{v = \frac{s}{t}} \leftarrow \text{distance } (s = \text{arc length})$$

velocity (speed)

Definition – Angular speed: – the angle swept out by an object over time

you are familiar with revolutions per minute rpm's

$$\boxed{\omega = \frac{\theta}{t}} \leftarrow \text{measured in radians}$$

omega

angular speed

Relationship b/t v + ω

$v = \frac{s}{t}$ but $s = r\theta$

$$v = \frac{r\theta}{t} \leftarrow \omega$$

$$\boxed{v = r\omega}$$

Example 8. A child is spinning a rock at the end of a 2-foot rope at the rate of 180 revolutions per minute (rpm). Find the linear speed of the rock when it is released.

$V = ?$

$r = 2\text{ ft.}$

$V = r\omega$

$\omega = \dfrac{180\text{ rev.}}{\text{min.}} = \dfrac{360\pi\text{ radian}}{\text{min.}}$

$\dfrac{180\text{ rev.}}{\text{min}} \cdot \dfrac{2\pi\text{ radian}}{1\text{ rev.}}$

$V = 2(360\pi)$

$V = 720\pi$

$V = 2{,}262\ \dfrac{\text{ft}}{\text{min}}$

$\dfrac{2{,}262\text{ ft.}}{\text{min}} \cdot \dfrac{1\text{ mile}}{5{,}280\text{ ft}} \cdot \dfrac{60\text{ min}}{1\text{ hr}} \approx 25.7\text{ mph}$

The linear speed of the rock when released is about 25.7 mph.