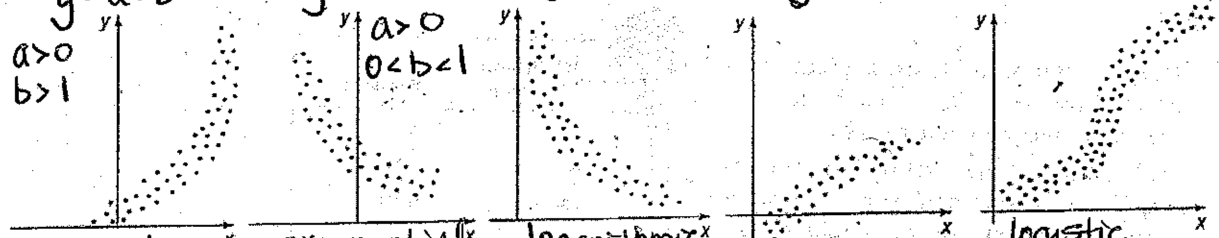
5.9 Building Exponential, Logarithmic, and Logistic Models from Data

Example scatterplots:

$y = a \cdot b^x$ $y = a \cdot b^x$ $y = a + b \ln x$ $y = a + b \ln x$ $y = \frac{c}{1 + ae^{-bx}}$

$a > 0$, $b > 1$ $a > 0$, $0 < b < 1$



exponential growth   exponential decay   logarithmic decay   logarithmic growth   logistic growth   logistic decay

Example 1: Kathleen is interested in finding a function that explains the growth of cell phone usage in the United States. She gathers data on the number (in millions) of U.S. cell phone subscribers from 1985 through 2005. The data are shown in the table below.

a) Using a graphing utility, draw a scatter diagram with year as the independent variable.
b) Using a graphing utility, build an exponential model from the data.
c) Express the function found in part b in the form $A = A_0 e^{kt}$
d) Graph the exponential function found on the scatter diagram.
e) Using the solution to part b or c, predict the number of U.S. cell phone subscribers in 2009.
f) Interpret the value of k found in part c.

Years since 1984 Year, x	Number of Subscribers (in millions), y
1985 ($x = 1$)	0.34
1986 ($x = 2$)	0.68
1987 ($x = 3$)	1.23
1988 ($x = 4$)	2.07
1989 ($x = 5$)	3.51
1990 ($x = 6$)	5.28
1991 ($x = 7$)	7.56
1992 ($x = 8$)	11.03
1993 ($x = 9$)	16.01
1994 ($x = 10$)	24.13
1995 ($x = 11$)	33.76
1996 ($x = 12$)	44.04
1997 ($x = 13$)	55.31
1998 ($x = 14$)	69.21
1999 ($x = 15$)	86.05
2000 ($x = 16$)	109.48
2001 ($x = 17$)	128.37
2002 ($x = 18$)	140.77
2003 ($x = 19$)	158.72
2004 ($x = 20$)	182.14
2005 ($x = 21$)	207.90
2006 ($x = 22$)	233.00

a) The data suggest an exponential growth model. The rate of change is not constant and there does appear to be a maximum or minimum. It is increasing over the domain and has no x-intercept. Additionally the rate of change appears to be increasing at a constant rate - a unique characteristic of exponentials.

b) $\boxed{y = .7267344773 \cdot (1.346976839)^x}$

$y = a \cdot b^x$

* c) $A = A_0 e^{kt}$ $A_0 = a$ $b^x = e^{kt}$

$\boxed{A = .7267344773 e^{\ln(1.346976839)t}}$ $x = t$ $b = e^k$

$1.346976839 = e^k$

$\ln(1.346976839) = k$

$k \approx .2978627028$

d) ✓

e) in 2009 $t = 25$

In 2009, the model predicts there will 1,245,601,059 cell phone subscribers in the U.S. This is completely unrealistic because there were only 306.8 million people in the United States.

f) k is the exponential growth rate. It tells me that the model predicts the number of cell phone subscripers is increasing 30% per year.

Example 2:

Jodi, a meteorologist, is interested in finding a function that explains the relation between the height of a weather balloon (in kilometers) and the atmospheric pressure (measured in millimeters of mercury) on the balloon. She collects the data shown in Table 11.

(a) Using a graphing utility, draw a scatter diagram of the data with atmospheric pressure as the independent variable.

(b) It is known that the relation between atmospheric pressure and height follows a logarithmic model. Using a graphing utility, build a logarithmic model from the data.

(c) Draw the logarithmic function found in part (b) on the scatter diagram.

(d) Use the function found in part (b) to predict the height of the weather balloon if the atmospheric pressure is 560 millimeters of mercury.

Table 11

Atmospheric Pressure, p	Height, h
760	0
740	0.184
725	0.328
700	0.565
650	1.079
630	1.291
600	1.634
580	1.862
550	2.235

a) STAT → EDIT – did table

ZOOM → #9 ZOOMSTAT

b) STAT → CALC → 9: LnReg

$y = 45.78632063 + -6.902524298 \ln x$

c) ✓ looks great!

d) $y = 45.78632063 - 6.902524298 \ln 560$

$y \approx 2.107583224$

When the atmospheric pressure is 560 mm Hg, then the height of the balloon is about 2.108 km.

Example 3: The data in the table below represent the amount of yeast biomass in a culture after t hours.

Time (in hours)	Yeast g Biomass	Time (in hours)	Yeast g Biomass
0	9.6	10	513.3
1	18.3	11	559.7
2	29.0	12	594.8
3	47.2	13	629.4
4	71.1	14	640.8
5	119.1	15	651.1
6	174.6	16	655.9
7	257.3	17	659.6
8	350.7	18	661.8
9	441.0		

Source: Tor Carlson (Über Geschwindigkeit und Grösse der Hefevermehrung in Würze, Biochemische Zeitschrift, Bd. 57, pp. 313–334, 1913)

(a) Using a graphing utility, draw a scatter diagram of the data with time as the independent variable. STAT ⟶ EDIT

(b) Using a graphing utility, build a logistic model from the data. STAT → CALC → B: Logistic

(c) Using a graphing utility, graph the function found in part (b) on the scatter diagram. ✓ looks good.

(d) What is the predicted carrying capacity of the culture?

(e) Use the function found in part (b) to predict the population of the culture at $t = 19$ hours.

b) $$y = \frac{c}{1+ae^{-bt}}$$

$$y = \frac{663.0219908}{1+71.57629487e^{-.546994726t}}$$

d) The model predicts the carrying capacity is about 663.02 g.

e) $t=19$

The model predicts the population will be about 661.57 grams after 19 hours.