5.7 Financial Models

Interest = \$ paid for use of money

Simple Interest Formula - $I = Prt$

I = simple

P = Principle (\$ borrowed/invested)

r = annual interest rate, given as %, use in decimal form)

t = time (years)

Compound Interest – interest paid on principle and previously earned interest

~~Example 1:~~ A credit union pays interest of 8% per annum compounded quarterly on a certain savings plan. If \$1000 is deposited in such a plan and the interest is left to accumulate, how much is in the account after 1 year.

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Compound Interest Formula - $A = P\left(1+\frac{r}{n}\right)^{nt}$

	n
annually	1
semi-annually	2
quarterly	4
monthly	12
daily	365

banks have 360

A = Account balance
Future value

P = Principle
Present value

n = # of times interest is compounded per year

P $\quad$ $r = .1$

Example 2: Investing $1000 at an annual rate of 10% compounded annually, semiannually, quarterly, monthly, and daily will yield the following amounts after 1 year. $\rightarrow t = 1$

$n = 1 \qquad A = 1000\left(1 + \frac{.1}{1}\right)^{1 \cdot 1} = 1000(1.1) = \$1{,}100$

$n = 2 \qquad A = 1000\left(1 + \frac{.1}{2}\right)^{2 \cdot 1} = \$1{,}102.50$

$n = 4 \qquad A = 1000\left(1 + \frac{.1}{4}\right)^{4 \cdot 1} = \$1{,}103.81$

$n = 12 \qquad A = 1000\left(1 + \frac{.1}{12}\right)^{12 \cdot 1} = \$1{,}104.71$

$n = 365 \qquad A = 1000\left(1 + \frac{.1}{365}\right)^{365} = \$1{,}105.16$

Continuous Compounding – $A = Pe^{rt}$

as $n \to \infty$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

Example 3: The amount A that results from investing a principal P of $1000 at an annual rate r of 10% compounded continuously for a time t of 1 year is

$A = Pe^{rt} \qquad\qquad P = 1000$

$A = 1000e^{.1} \qquad\qquad r = .1$

$A = \$1{,}105.17 \qquad\qquad t = 1$

The account balance will be $1,105.17 after 1 year.

Effective Rate of Interest -

(allows you to compare different options)

① Compounding n times per year

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1$$

② compounding continuously

$$r_e = e^r - 1$$

r_e = equivalent annual simple interest rate.

Example 4: Suppose you want to open a money market account. You visit three banks to determine their money market rates. Bank A offers you 6% annual interest compounded daily and Bank B offers you 6.02% compounded quarterly. Bank C offers 5.98% compounded continuously. Determine which bank is offering the best deal.

Bank A

$r = .06 \quad n = 365$

$r_e = \left(1+\frac{r}{n}\right)^n - 1$

$r_e = \left(1+\frac{.06}{365}\right)^{365} - 1$

$r_e \approx .0618$

$r_e \approx 6.18\%$

Bank B

$r = .0602 \quad n = 4$

$r_e = \left(1+\frac{.0602}{4}\right)^4 - 1$

$r_e \approx .0616$

$r_e \approx 6.16\%$

Bank C

$r = .0598$

$r_e = e^r - 1$

$r_e = e^{.0598} - 1$

$r_e \approx .0616$

$r_e \approx 6.16\%$

Because Bank A has the highest effective interest, they are offering the best deal.

Present ~~Value~~ Formulas –

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Example 5: A zero-coupon (noninterest-bearing) bond can be redeemed in 10 years for \$1000. How much should you be willing to pay for it now if you want a return of
(a) 8% compounded monthly?
(b) 7% compounded continuously?

a) $t = 10 \quad r = .08 \quad n = 12$

$A = 1000$

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$1000 = P\left(1+\frac{.08}{12}\right)^{12(10)}$

$\dfrac{1000}{\left(1+\frac{.08}{12}\right)^{120}} = P$

$450.52 = P$

I would be willing to buy the bond at \$450.52 for a return of 8% compounded monthly.

b) $A = Pe^{rt}$

$\dfrac{1000}{e^{.7}} = \dfrac{Pe^{.07(10)}}{e^{.7}}$

$\dfrac{1000}{e^{.7}} = P$

$\$496.59 = P$

For a return of 7% compounded continuously, I would be willing to purchase the bond for \$496.59.

Example 6: What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years? $P = P \quad A = 2P \quad t = 5 \quad n = 1 \quad r = ?$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$2P = P\left(1+\frac{r}{1}\right)^{1 \cdot 5}$$

$$\frac{2\cancel{P}}{\cancel{P}} = \frac{\cancel{P}(1+r)^5}{\cancel{P}}$$

$$2 = (1+r)^5$$

$$\sqrt[5]{2} = \sqrt[5]{(1+r)^5}$$

$$\sqrt[5]{2} = 1+r$$

$$\sqrt[5]{2} - 1 = r$$

$$.1487 \approx r$$

If I want to double my investment after 5 years, I should seek an annual interest of 14.87 %.

Example 7: (a) How long will it take for an investment to double in value if it earns 5% compounded continuously?
(b) How long will it take to triple at this rate?

a) $A = 2P \quad r = .05 \quad t = ?$

b)

$$A = Pe^{rt}$$

$$\frac{2P}{P} = \frac{Pe^{.05t}}{P}$$

$$2 = e^{.05t}$$

$$\ln 2 = .05t$$

$$\frac{\ln 2}{.05} = t$$

$$13.86 = t$$

To double an investment earning 5% compounded continuously, it would take a little under 14 years.