

5.6 Logarithmic and Exponential Equations

Example 1: Solve: $2\log_5 x = \log_5 9$

$$\{x | x > 0\}$$

$$\log_5 x^2 = \log_5 9$$

$$x^2 = 9$$

$$\sqrt{x^2} = \pm \sqrt{9}$$

$$x = \pm 3$$

-3 < 0 extraneous

$$\boxed{x = 3}$$

① Determine domain

② Solve log equation

③ Check for extraneous solutions

Example 2: Solve $\log_5(x+6) + \log_5(x+2) = 1$

$$x+6 > 0$$

$$x+2 > 0$$

$$x > -6$$

$$x > -2$$

take more

restrictive domain

$$\{x | x > -2\}$$

$$\log_5(x+6) + \log_5(x+2) = 1$$

$$\log_5[(x+6)(x+2)] = 1$$

$$\log_5[x^2 + 8x + 12] = 1$$

$$5^1 = x^2 + 8x + 12$$

$$x^2 + 8x + 7 = 0$$

$$(x+1)(x+7) = 0$$

$$x+1=0 \text{ or } x+7=0$$

$$\boxed{x = -1} \text{ or } \cancel{x = -7}$$

$$-7 < -2$$

$$\frac{1 \cdot 7}{1+7} = 7$$

$$\frac{1 \cdot 7}{1+7} = 8$$

Example 3: Solve $\ln x + \ln(x-4) = \ln(x+6)$

$$x > 0$$

$$x > 4$$

$$x > -6$$

$$\{x | x > 4\}$$

$$\ln x + \ln(x-4) = \ln(x+6)$$

$$\ln[x(x-4)] = \ln(x+6)$$

$$x(x-4) = x+6$$

$$x^2 - 4x = x+6$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$x-6=0$$

$$\boxed{x = 6}$$

$$x+1=0$$

$$\cancel{x = -1}$$

$$-1 < 4$$

No need to worry about domain

Example 4: Solve: $2^x = 5$

$$\log_2 2^x = \log_2 5$$

$$x = \log_2 5$$

$$x = \frac{\ln 5}{\ln 2} = \frac{\log 5}{\log 2} \approx 2.3219$$

exact
solution

Example 5: Solve: $\frac{8 \cdot 3^x}{8} = \frac{5}{8}$

$$3^x = \frac{5}{8}$$

$$\log_3 \left(\frac{5}{8} \right) = x$$

$$x = \frac{\ln \left(\frac{5}{8} \right)}{\ln 3} = \frac{\log \left(\frac{5}{8} \right)}{\log 3} \approx -0.4278$$

exact approximation

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Example 6: $5^{x-2} = 3^{3x+2}$

$$\ln 5^{x-2} = \ln 3^{3x+2}$$

$$(x-2)\ln 5 = (3x+2)\ln 3$$

$$x\ln 5 - 2\ln 5 = 3x\ln 3 + 2\ln 3$$

$$x\ln 5 - 3x\ln 3 = 2\ln 3 + 2\ln 5$$

$$\frac{x(\ln 5 - 3\ln 3)}{\ln 5 - 3\ln 3} = \frac{2\ln 3 + 2\ln 5}{\ln 5 - 3\ln 3}$$

$$x = \frac{2\ln 3 + 2\ln 5}{\ln 5 - 3\ln 3}$$

$$x = \frac{\ln 3^2 + \ln 5^2}{\ln 5 - \ln 3^3} = \frac{\ln 9 + \ln 25}{\ln 5 - \ln 27} = \frac{\ln(9 \cdot 25)}{\ln \left(\frac{5}{27} \right)} = \frac{\ln 225}{\ln \left(\frac{5}{27} \right)}$$

$$\approx -3.2116$$

Example 7: $4^x - 2^x - 12 = 0$ will want to use "u substitution"

$$(2^x)^2 - 2^x - 12 = 0$$

$$u = ?$$

$$2^{2x} - 2^x - 12 = 0$$

$$2^{x \cdot 2} - 2^x - 12 = 0$$

$$u = 2^x$$

$$(2^x)^2 - 2^x - 12 = 0$$

$$\frac{-4 \pm 3}{-4 \pm 3} = -1$$

$$u^2 - u - 12 = 0$$

$$(u-4)(u+3) = 0$$

$$u-4=0 \quad u+3=0$$

$$u=4 \quad u=-3$$

$$2^x = 4$$

$$\boxed{x=2}$$

$$\cancel{2^x = -3}$$

no solution

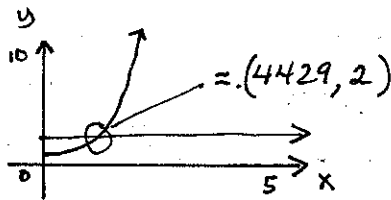
Example 8: $x + e^x = 2$

problem - x is down and in exponent position
- can't solve algebraically

use graphing calc.

$$Y_1 = x + e^x$$

$$Y_2 = 2$$



use "intersect" to find x

$$x \approx 4.429$$