5.5 Properties of Logarithms "Logarithm Rules"

Example 1: "Prove" Establishing Properties of Logarithms

a) Show that $\log_a 1 = 0$

$\log_a 1 = y$

$a^y = 1$

$a^y = a^0$

$y = 0 \quad \therefore \log_a 1 = 0$

b) Show that $\log_a a = 1$

$\log_a a = y$

$a^y = a$

$a^y = a^1$

$y = 1$

$\therefore \log_a a = 1$

Properties of Logarithms:

$\log_a 1 = 0 \quad \log_a a = 1$

$\log_a a^r = r \quad a^{\log_a M} = M$

$\log_a M^r = r \log_a M \Leftarrow$ proof pg. 299

$\log_a (MN) = \log_a M + \log_a N$

$\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

$M = N$ iff $\log_a M = \log_a N$

Example 2: Using the above properties

a) $2^{\log_2 \pi} = \pi$

b) $\log_{0.2} 0.2^{-\sqrt{2}} = -\sqrt{2}$

c) $\ln e^{kt} = kt$

possible

Example 3: Write $\log_a(x\sqrt{x^2+1})$, x > 0, as a sum of logarithms. Express all powers as factors.

$$\log_a(x\sqrt{x^2+1}) = \log_a x + \log_a\sqrt{x^2+1}$$
$$= \log_a x + \log_a (x^2+1)^{\frac{1}{2}}$$
$$= \boxed{\log_a x + \tfrac{1}{2}\log_a (x^2+1)}$$

Example 4: Write

$$\ln\frac{x^2}{(x-1)^3} \quad , \qquad x > 1$$

As a difference of logarithms. Express all powers as factors.

$$\ln\left(\frac{x^2}{(x-1)^3}\right) = \ln x^2 - \ln (x-1)^3$$
$$= 2\ln x - 3\ln(x-1)$$

Example 5: Write

$$\log_a\frac{\sqrt{x^2+1}}{x^3(x+1)^4} \quad , \qquad x > 0$$

as a sum and difference of logarithms. Express all powers as factors.

$$\log_a \frac{\sqrt{x^2+1}}{x^3(x+1)^4} = \log_a\sqrt{x^2+1} - \log_a(x^3(x+1)^4)$$
$$= \log_a\sqrt{x^2+1} - \left[\log_a x^3 + \log_a (x+1)^4\right]$$
$$= \log_a (x^2+1)^{\frac{1}{2}} - \left[\log_a x^3 + \log_a (x+1)^4\right]$$
$$= \frac{1}{2}\log_a(x^2+1) - \left[3\log_a x + 4\log_a (x+1)\right]$$
$$= \frac{1}{2}\log_a(x^2+1) - 3\log_a x - 4\log_a(x+1)$$

Example 6: Write each of the following as a single logarithm. Express all factors (possible) as powers.

a) $\log_a 7 + 4\log_a 3 = \log_a 7 + \log_a 3^4$

$= \log_a (7 \cdot 3^4)$

$= \log_a (7 \cdot 81)$

$= \log_a (567)$

b) $\frac{2}{3}\ln 8 - \ln(3^4 - 8) = \ln 8^{\frac{2}{3}} - \ln(3^4 - 8)$

$= \ln (\sqrt[3]{8})^2 - \ln(81 - 8)$

$= \ln 4 - \ln 73$

$= \ln \frac{4}{73}$

c) $\underbrace{\log_a x + \log_a 9}_{} + \log_a(x^2 + 1) - \log_a 5 = \underbrace{\log_a 9x + \log_a (x^2+1)}_{} - \log_a 5$

$= \log_a (9x(x^2+1)) - \log_a 5$

$= \log_a \left[\frac{9x(x^2+1)}{5}\right]$

Example 7: Approximate $\log_2 7$. Round your answer to four decimal places.

$$\log_2 7 = y$$
$$2^y = 7$$
$$\ln 2^y = \ln 7$$
$$y \ln 2 = \ln 7$$
$$y = \frac{\ln 7}{\ln 2} \approx 2.8073$$

Change of Base Formula

$$\log_a M = y$$
$$a^y = M$$
$$\log_b a^y = \log_b M$$
$$y \log_b a = \log_b M$$
$$y = \frac{\log_b M}{\log_b a}$$

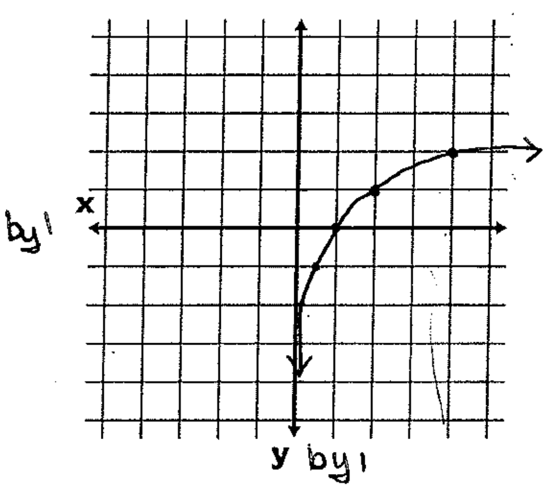
$$\therefore \boxed{\log_a M = \frac{\log_b M}{\log_b a}} = \frac{\ln M}{\ln a} = \frac{\log M}{\log a}$$

Example 8: Approximate

a) $\log_5 89 = \frac{\ln 89}{\ln 5} = \frac{\log 89}{\log 5} \approx 2.7889$

b) $\log_{\sqrt{2}} \sqrt{5} = \frac{\ln \sqrt{5}}{\ln \sqrt{2}} = \frac{\log \sqrt{5}}{\log \sqrt{2}}$

Example 9: Graph $y = \log_2 x$ on your graphing calculator.



$$Y_1 = \frac{\ln x}{\ln 2} = \frac{\log x}{\log 2}$$

x	y
$\frac{1}{2}$	-1
1	0
2	1