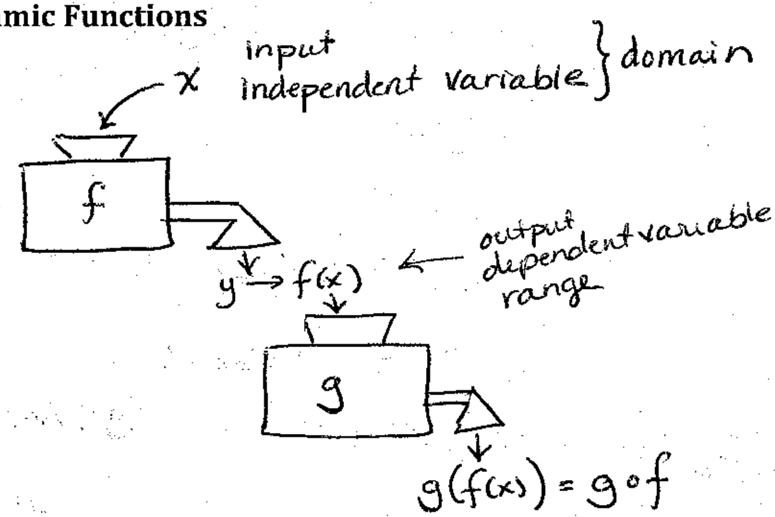
Chapter 5 Exponential and Logarithmic Functions

5.1 Composite Functions



Definition of composition –
Given 2 functions f and g, the composite function, denoted $f \circ g$ (read as f composed with g) is defined as $(f \circ g)(x) = f(g(x))$

Example 1: Evaluating a Composite Function

Suppose that $f(x) = 2x^2 - 3$ and $g(x) = 4x$. Find:

(a) $(f \circ g)(1)$

$= f(g(1))$
$= f(4 \cdot 1)$
$= f(4)$
$= 2(4)^2 - 3$
$= 32 - 3$
$(f \circ g)(1) = 29$

(b) $(g \circ f)(1)$

$= g(f(1))$
$= g(2(1)^2 - 3)$
$= g(-1)$
$= 4(-1)$
$= -4$
$(g \circ f)(1) = -4$

order matters
~ not commutative

(c) $(f \circ f)(-2)$

$= f(f(-2))$
$= f(2(-2)^2 - 3)$
$= f(5)$
$= 2(5)^2 - 3$
$= 50 - 3$
$(f \circ f)(-2) = 47$

(d) $(g \circ g)(-1)$

$= g(g(-1))$
$= g(4 \cdot -1)$
$= g(-4)$
$= 4(-4) = -16$
$(g \circ g)(-1) = -16$

2 approaches
① Find domain BEFORE you find composite
② Find domain AFTER you find composite

Example 2: Finding a Composite Function and Its Domain

D of f: $\mathbb{R}$ D of g: $\mathbb{R}$

Suppose that $f(x) = x^2 + 3x - 1$ and $g(x) = 2x + 3$. Find: (and then find the domain of each)

(a) $(f \circ g)(x) = f(g(x))$

$= f(2x+3)$

$= (2x+3)^2 + 3(2x+3) - 1$

$= 4x^2 + 12x + 9 + 6x + 9 - 1$

$(f \circ g)(x) = 4x^2 + 18x + 17$

D: $\mathbb{R}$

(b) $(g \circ f) = g(f(x))$

$= g(x^2 + 3x - 1)$

$= 2(x^2 + 3x - 1) + 3$

$= 2x^2 + 6x - 2 + 3$

$(g \circ f)(x) = 2x^2 + 6x + 1$

D: $\mathbb{R}$

Example 3: Finding the Domain of $(f \circ g)$ (find before finding $(f \circ g)$)

Find the domain of $(f \circ g)(x)$ if $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{4}{x-1}$.

D of f: $\{x \mid x \neq -2\}$ D of g: $\{x \mid x \neq 1\}$

no input of −2

Domain of $f(g(x))$

b/c g is 1st function - exclude its domain

so the output of g can't be −2

D of $(f \circ g)$: $\{x \neq 1, x \neq -1\}$

$g(x) = -2$

$\cancel{(x-1)} \frac{4}{\cancel{x-1}} = -2(x-1)$

$4 = -2x + 2$

$-2 \qquad -2$

$2 = -2x$

$-1 = x$

D of f $\{x \mid x \neq -2\}$ D of g $\{x \mid x \neq 1\}$

Example 4: Suppose that $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{4}{x-1}$. Find

(a) $(f \circ g)$

(b) $(f \circ f)$

Then find the domain of each composite function.

a) $(f \circ g)(x) = f(g(x))$

$= f\left(\frac{4}{x-1}\right)$

$= \dfrac{1}{\left(\frac{4}{x-1}\right) + 2}$

$= \dfrac{1}{\frac{4}{x-1} + \frac{2x-2}{x-1}}$

$= \dfrac{1}{\left(\frac{2x+2}{x-1}\right)}$

$= \dfrac{x-1}{2x+2}$

$= \dfrac{x-1}{2(x+1)}$

D: $\{x \mid x \neq 1, x \neq -1\}$

b) $(f \circ f)(x) = f(f(x))$

$= f\left(\frac{1}{x+2}\right)$

$= \dfrac{1}{\left(\frac{1}{x+2}\right) + 2}$

$= \dfrac{1}{\frac{1}{x+2} + \frac{2x+4}{x+2}}$

$= \dfrac{1}{\frac{2x+5}{x+2}}$

$= \dfrac{x+2}{2x+5}$

$2x+5 = 0$

$x = -\frac{5}{2}$

D: $\{x \mid x \neq -2, x \neq -\frac{5}{2}\}$

Example 5: Show that two composite functions are equal.

If $f(x) = 3x - 4$ and $g(x) = \frac{1}{3}(x+4)$, show that $(f \circ g)(x) = (g \circ f)(x) = x$ for every x in the domain of $(f \circ g)$ and $(g \circ f)$.

$$(f \circ g)(x) = f(g(x)) = f\left(\tfrac{1}{3}[x+4]\right) = 3\left[\tfrac{1}{3}(x+4)\right] - 4 = x+4-4 = x$$

for all x in the domain $g(x)$.

$$(g \circ f)(x) = g(f(x)) = g(3x-4) = \tfrac{1}{3}[3x-4+4] = \tfrac{1}{3}[3x] = x$$

for all x in the domain of $f(x)$.

Example 6: Finding the Components of Composite Function
Find functions f and g such that $(f \circ g) = H$ if $H(x) = (x^2+1)^{50}$

use order of operations to determine inner function

$(f \circ g)(x) = f(g(x))$

$g(x) = x^2 + 1$

$f(x) = x^{50}$

Check.

$f(g(x)) = f(x^2+1)$

$= (x^2+1)^{50}$ ✓

Example 7: Finding the Components of Composite Function
Find functions f and g such that $(f \circ g) = H$ if $H(x) = \frac{1}{x+1}$

$g(x) = x+1$

$f(x) = \frac{1}{x}$

Check

$f(g(x)) = f(x+1)$

$= \frac{1}{x+1}$ ✓