1.4 Notetaking with Vocabulary

***Learning target*: Understand solving liner equations.**

Success criteria: I can solve absolute-value equations

Write the meaning of each vocabulary term.

absolute value equation - an equation that contains an absolute value expression $|x| = 3$ $|y-6| = 6$

- solved by solving 2 related linear equations. $2x|x+2| - 9 = |x-3| + 2$

extraneous solution

- an apparent solution that must be rejected b/c it does not satisfy the original equation

Core Concepts

Properties of Absolute Value

Let a and b be real numbers. Then the following properties are true.

1. $|a| \geq 0$ non-negative

2. $|-a| = |a|$

3. $|ab| = |a||b|$

4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$

Solving Absolute Value Equations

To solve $|ax + b| = c$ when $c \geq 0$, solve the related linear equations

$$ax + b = c \quad \text{or} \quad ax + b = -c.$$

When negative $c < 0$, the absolute value equation $|ax + b| = c$ has no solution because absolute value always indicates a number that is not negative.

1.4 Solving Absolute Value Equations continued…

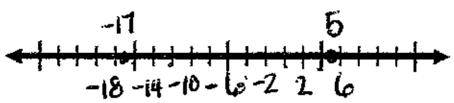
Example 1. Solve each equation. Graph the solutions if possible to a, b and c.

a. $|x+6| = 11$

$x + \cancel{6} = 11$ or $x + \cancel{6} = -11$

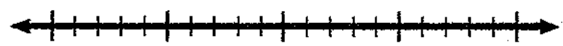
$\quad -\cancel{6} \quad -6 \qquad\qquad -\cancel{6} \quad -6$

$x = 5$ or $x = -17$



b. $|2x-1| = -4$

no solution



c. $|3x-6| - \cancel{9} = -3$

$\quad +9 \quad +9$

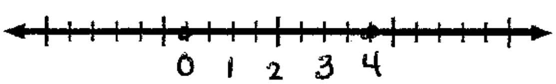
$|3x-6| = 6$

$3x - \cancel{6} = 6$ or $3x - 6 = -6$

$\quad +\cancel{6} \quad +6 \qquad\qquad +6 \quad +6$

$\frac{3x}{3} = \frac{12}{3} \qquad \frac{3x}{3} = \frac{0}{3}$

$x = 4$ or $x = 0$



Check:

$|3 \cdot 4 - 6| - 9 \stackrel{?}{=} -3$

$|12-6| - 9 \stackrel{?}{=} -3$

$|6| - 9 \stackrel{?}{=} -3$

$6 - 9 \stackrel{?}{=} -3$

$-3 = -3$

$|3 \cdot 0 - 6| - 9 \stackrel{?}{=} -3$

$|0-6| - 9 \stackrel{?}{=} -3$

$|-6| - 9 \stackrel{?}{=} -3$

$6 - 9 = -3$

$-3 = -3$ ✓

d. $-2|5x-1| - 3 = -11$

$\quad +3 \quad +3$

$\frac{\cancel{-2}|5x-1|}{\cancel{-2}} = \frac{-8}{-2}$

$|5x-1| = 4$

$5x - \cancel{1} = 4$ or $5x - \cancel{1} = -4$

$\quad +\cancel{1} \quad +1 \qquad\qquad +\cancel{1} \quad +1$

$\frac{5x}{5} = \frac{5}{5}$ or $\frac{5x}{5} = \frac{-3}{5}$

$x = 1$ or $x = \frac{-3}{5}$

Example 2: In a cheerleading competition, the minimum length of a routine is 4 minutes. The maximum length of a routine is 5 minutes. Write an absolute value equation that represents the minimum and maximum lengths.

$x = 5$ $\longrightarrow |ax+b| = c$

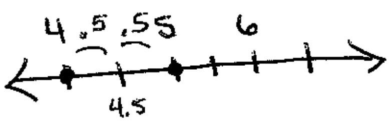
$x = 4$

to help?

graph

by $\frac{1}{2}$

halfway pt = 4.5



$a = 1$

$|x - 4.5| = .5$

Check !

Solving Equations with Two Absolute Values

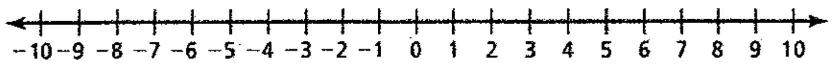
To solve $|ax + b| = |cx + d|$, solve the related linear equations — the opposite of

$$ax + b = cx + d \qquad or \qquad ax + b = -\underbrace{(cx + d)}_{\cdot(-1)}.$$

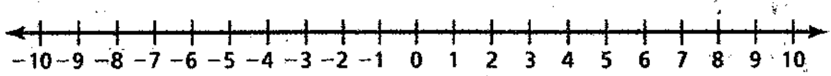
Notes:

In Examples 1–2, solve the equation. Graph the solution(s), if possible.

1. $|3x + 12| = 0$

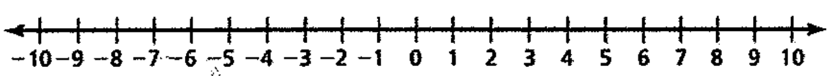


2. $-4|7 - 6k| = 14$

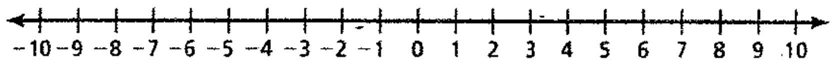


Practice: Solve the equation. Graph the solution(s), if possible.

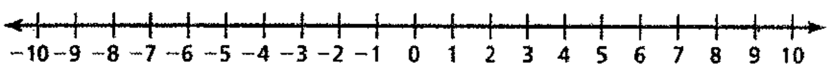
1. $|y + 2| = 8$



2. $\left|\frac{d}{3}\right| = 3$



3. $3|2x + 5| + 10 = 37$



Example 3 and 4. Solve the equation. Check your solutions.

3. $|20x| = |4x + 16|$

$20x = 4\cancel{x} + 16$ OR $20x = -1(4x + 16)$

$-4x \quad -4x$

$\frac{16x}{16} = \frac{16}{16}$

$\boxed{x = 1}$

$20x = -4\cancel{x} - 16$

$+4x \quad +4x$

$\frac{24x}{24} = \frac{-16}{24}$

$x = \frac{-16 \div 8}{24 \div 8} = \boxed{\frac{-2}{3}}$

4. $|p + 4| = |p - 2|$

$\cancel{p} + 4 = \cancel{p} - 2$ OR $p + 4 = -(p - 2)$

$-p \quad -p$

$4 = -2$

no solution

$p + 4 = -\cancel{p} + 2$

$+p \quad +p$

$2p + \cancel{4} = 2$

$-4 \quad -4$

$\frac{2p}{2} = \frac{-2}{2}$

$\boxed{p = -1}$